CSE 16 7-6-21

(2.8) Quantifiers & conditionals

In many math texts you'll see things like

$$x^2 - y^2 = (x-y)(x+y).$$

what the author means is

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x^2 - y^2 = (x-y)(x+y)$$

i.e. quantification is implicit.

(2.9) Translation

Consider the statement

"all horses in California are blue"

How to translate into 1st order logic.

Ex. $U = \{\text{horses in California}\}$

$B(x) =$ " x is blue"

Translation: $\forall x \; B(x)$

Ex. $U = \{\text{horses}\}$

$C(x) =$ " x lives in CA "

Translation: $\forall x \; (C(x) \Rightarrow B(x))$

Ex. $U = \{\text{living things}\}$

$H(x) =$ " x is a horse"

<u>Translation</u>: $\forall x \, (H(x) \wedge C(x) \Rightarrow B(x))$

<u>note</u>: $T \Rightarrow P \equiv P$ since

P	$T \Rightarrow P$
0	0
1	1

same !!

so first example could be :

$$\forall x \, (T \Rightarrow B(x))$$

In general, if $S \subseteq \mathcal{U}$, then

$$\forall x \in \mathcal{U} \ (x \in S \Rightarrow P(x))$$

is same as

$$\forall x \in S : P(x)$$

<u>Ex.</u> $\mathcal{U} = \{ \text{People} \}$

$L(x, y) =$ " x loves y "

a.) " everybody loves somebody "

$$\forall x \, \exists y : L(x, y)$$

b.) "somebody loves everybody"

$$\exists x\ \forall y : L(x, y)$$

c.) $\exists y\ \forall x : L(x, y)$

"there is somebody whom everybody loves"

d.) $\forall y\ \exists x : L(x, y)$

"everybody is loved by somebody"

e.) $\sim \exists x\ \forall y : L(x, y)$

"nobody loves everybody"

f.) "there is exactly one person who everyone loves"

$$\exists y \left(\forall x\, L(x,y) \wedge \forall z \left(z \neq y \Rightarrow \exists w \sim L(w,z) \right) \right)$$

g.) "there is a person who loves everyone but themself"

$$\exists x \left(\sim L(x,x) \wedge \forall y \left(y \neq x \Rightarrow L(x,y) \right) \right)$$

<u>Ex.</u> $U = \{\text{people}\}$

$E(x) =$ " x is the king of England "

$F(x) =$ " x is the king of France "

<u>Translate</u>: "no king of England is not the king of France."

$$\sim \exists x \, (E(x) \wedge \sim F(x))$$

2.10 Negations

Consider a finite universe

$$U = \{1, 2\}$$

Let $P(x)$ be a prop. fcn. on U.

Then

$$\forall x \, P(x) = P(1) \wedge P(2)$$

and

$$\exists x \, P(x) = P(1) \vee P(2)$$

Thus

$$\sim \forall x P(x) = \sim (P(1) \wedge P(2))$$

$$= \sim P(1) \vee \sim P(2)$$

$$= \exists x \sim P(x)$$

and

$$\sim \exists x P(x) = \sim (P(1) \vee P(2))$$

$$= \sim P(1) \wedge \sim P(2)$$

$$= \forall x \sim P(x)$$

Identities (for any universe U):

$$\sim \forall x P(x) = \exists x \sim P(x)$$

$$\sim \exists x P(x) = \forall x \sim P(x)$$

Thus

stmt.	true when	false when
$\forall x\, P(x)$	$P(x)$ is true for all $x \in U$	$P(x)$ is false for at least one $x \in U$
$\exists P(x)$	$P(x)$ is true for at least one $x \in U$	$P(x)$ is false for all $x \in U$

<u>Ex.</u> England/France again.

"no King of England is not the King of France"

$$\sim \exists x\, (E(x) \wedge \sim F(x))$$

$$= \forall x \sim (E(x) \wedge \sim F(x))$$

$$= \forall x \ (\sim E(x) \vee \sim\sim F(x))$$

$$= \forall x \ (\sim E(x) \vee F(x))$$

$$= \forall x \ (E(x) \Rightarrow F(x))$$ by the implication law

2.11
2.12 read these

Chapter 3: Combinatorics

(3.1) lists

<u>Defn</u>

a <u>list</u> (or <u>sequence</u>) is an ordered collection of objects, where repetition is allowed.

<u>notation</u>: $(1, 3, 2, 2, 1)$

<u>observe</u>: a list of <u>length</u> n is the same as an ordered n-tuple.

The objects in a list are called: <u>terms</u>, <u>entries</u>, <u>components</u>, <u>elements</u>.

Empty List: $( ) = \phi$

$\uparrow$ occasionally

Defn

A string is a sequence of symbols from some alphabet, written without commas or parentheses.

Ex. list of length 5:

$$(1, 3, 2, 2, 1)$$

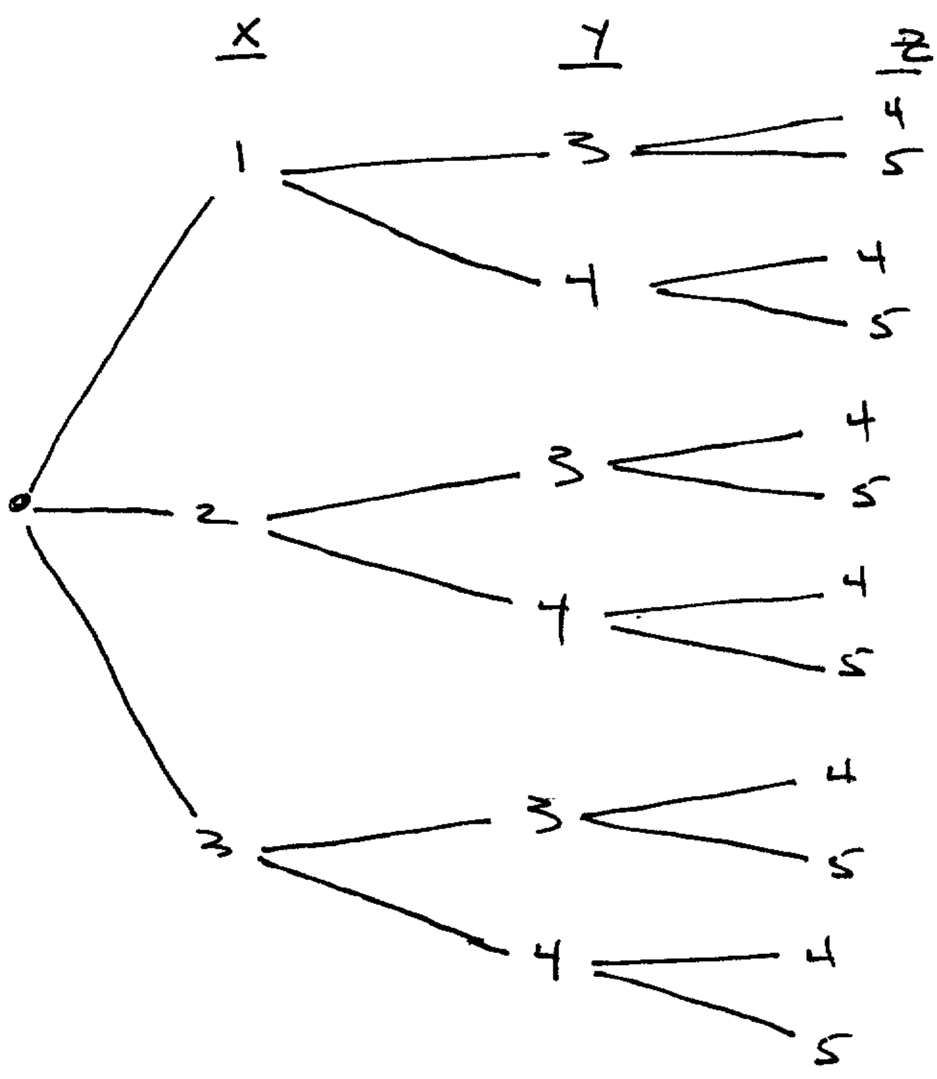
string of length 5:

$$13221 \quad \text{alphabet} = \{1, 2, 3\}$$

(3.2) Multiplication Principle

Ex. find the number of lists (x, y, z) where $x \in \{1, 2, 3\}$, $y \in \{3, 4\}$, $z \in \{4, 5\}$

tree diagram



list
(1, 3, 4)
(1, 3, 5)
(1, 4, 4)
(1, 4, 5)
(2, 3, 4)
(2, 3, 5)
(2, 4, 4)
(2, 4, 5)
(3, 3, 4)
(3, 3, 5)
(3, 4, 4)
(3, 4, 5)

(# lists) $= \boxed{12} = 3 \cdot 2 \cdot 2$

In general

$$|\{(x,y,z) \mid x\in A \wedge y\in B \wedge z\in C\}|$$

$$= |A\times B\times C| = |A|\cdot|B|\cdot|C|$$

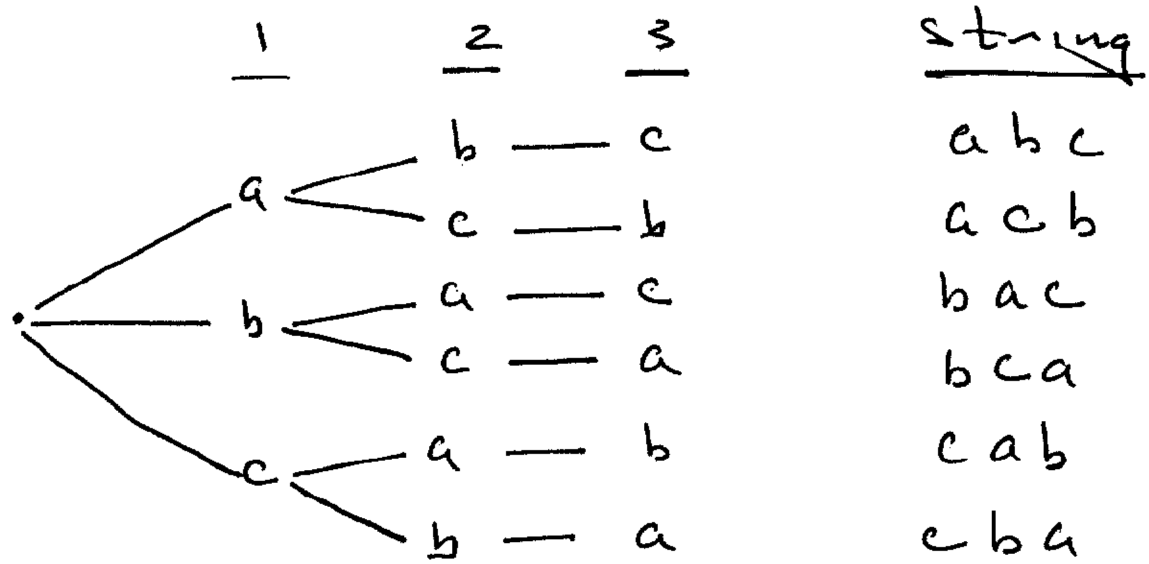
Ex. find # of strings of length 3 from alphabet $\{a,b,c\}$

$$\begin{array}{ccc} 1 & 2 & 3 \\ \square & \square & \square \end{array}$$

#choices $= 3\cdot 3\cdot 3 = 3^3 = \boxed{27}$

Ex. find the # of such strings in which no letter is repeated.

tree diagram



$\#$ strings $= \boxed{6} = 3 \cdot 2 \cdot 1$

Multiplication Principle

suppose in forming a list

$$(x_1, x_2, \ldots, x_n)$$

of length n, there are

a_1 choices for x_1

a_2 " " x_2

a_3 " " x_3

$\vdots$

a_n " " x_n

Then the number of such lists is $\boxed{a_1 \cdot a_2 \cdot a_3 \cdots a_n}$

<u>note</u>: The # choices a_i for x_i may depend on the preceding choices

$$(\underbrace{x_1, x_2, \ldots, x_{i-1}}, x_i, \ldots\ldots\ldots)$$

may depend on