CSE 16 7-1-21

There is another way to prove set identities : <u>membership table</u>

let A, B, C be sets. let

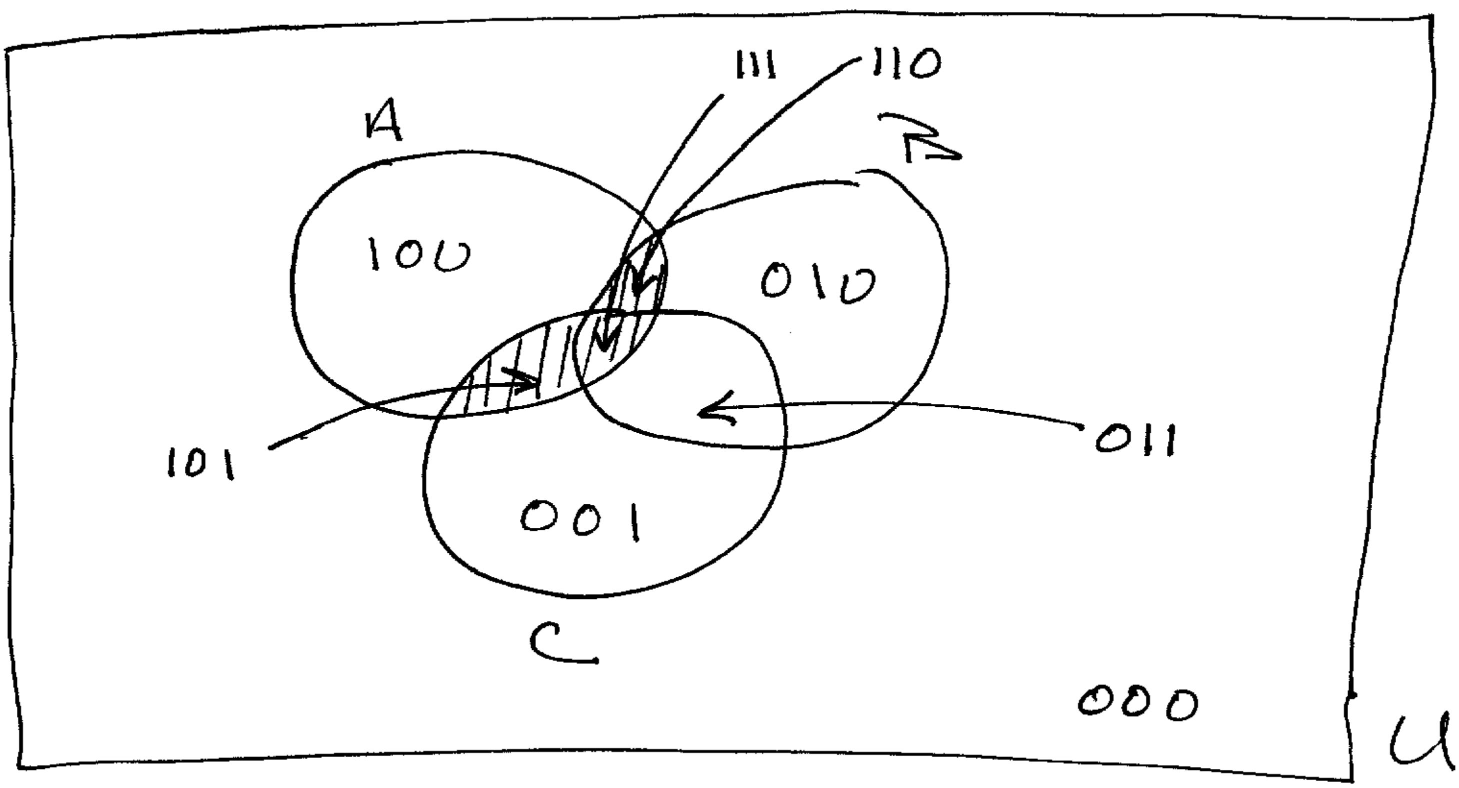
$$P = (x \in A)$$
$$Q = (x \in B)$$
$$R = (x \in C)$$

<u>Ex.</u> Prove 1st dist. law

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \checkmark$$

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

same



Abstract Algebraic System:

Boolean Algebra

- set theory
- propositional logic
- circuits

Duality (in logic)

Let X be a compound statement containing only ops: $\wedge, \vee, \sim$

constants: T, F

The *dual* of X is obtained by swaping:

$$\wedge \Leftrightarrow \vee$$

$$T \Leftrightarrow F$$

<u>Theorem</u>

If $X = Y$ then $\text{dual}(X) = \text{dual}(Y)$.

<u>Ex</u> what is dual of $P \Rightarrow Q$?

$$P \Rightarrow Q = \sim P \vee Q$$

$$\begin{aligned}
\text{dual}(P \Rightarrow Q) &= \text{dual}(\sim P \vee Q) \\
&= \sim P \wedge Q \\
&= \sim (P \vee \sim Q) \\
&= \sim (\sim Q \vee P) \\
&= \sim (Q \Rightarrow P)
\end{aligned}$$

(2.7) Quantifiers

logic of statements is called:

- Propositional logic
- sentential logic
- 0^{th} order logic

Enlarged system of logic:

- 1^{st} order logic
- Predicate logic
- Quantifier logic

<u>Recall</u>: A <u>Propositional Function</u> (Predicate, open sentence) $P(x)$ is a sentence that becomes a statement when a value for x is specified.

Ex $P(x) = (x < 7)$

$P(2) = (2 < 7)$ T

$P(10) = (10 < 7)$ F

Ex. $P(x) =$ 'x is a sunny day'

$P(\text{today}) =$ 'today is a sunny day'

Ex $Q(x, y) = (x + y = 10)$

$Q(3, 7) = (3 + 7 = 10)$ T

$Q(3, 8) = (3 + 8 = 10)$ F

The Universe of discourse or the Domain of a propositional fcn. is the set U of values that can be substituted for its variable(s).

Another way to obtain a statement from a Prop. fcn. $P(x)$ is through <u>Quantification</u>.

- Universal Quantifier: $\forall$ (for all)
- Existential Quantifier: $\exists$ (there exists) (for some)

<u>Defn</u>

- $\forall x \in U : P(x)$

  $\forall x \, P(x)$

  meaning: 'for all $x \in U$, $P(x)$ is true.'

- $\exists x \in \mathcal{U} : P(x)$

  $\exists x \; P(x)$

  meaning:

  'there exists an $x \in \mathcal{U}$ such that $P(x)$ is true'

  'there exists at least one $x \in \mathcal{U}$ such that $P(x)$ is true'

  'for some $x \in \mathcal{U}$, $P(x)$ is true'

Ex. $P(x) = (x^2 \geq 0) \qquad \mathcal{U} = \mathbb{R}$

$\forall x \, P(x) =$ 'the square of any real number is non-negative'

$\exists x \, P(x) =$ 'the square of some real number is non-negative.'

Ex. $Q(x) = (x < 50)$ $\quad U = \mathbb{R}$

$\forall x\, Q(x) =$ 'every real number is less than 50'

$\exists x\, Q(x) =$ 'there exists at least one real number that is less than 50.'

Ex. $R(x) = (x^2 < 0)$ $\quad U = \mathbb{R}$

$\forall x\, R(x) =$ 'the square of every real number is negative'

$\exists x\, R(x) =$ 'there is a real number whose square is negative'

Ex: $R(x) = (x^2 < 0)$ $\quad U = \mathbb{C}$

$\exists_x R(x) =$ 'there exists a complex number whose square is negative.'

note: $\mathbb{C} = \{x + iy \mid x \in \mathbb{R}, y \in \mathbb{R}\}$

defn $i^2 = -1$

so $i = 0 + 1 \cdot i \in \mathbb{C}$

so $i^2 = -1 < 0$.

Ex: $P(x,y) = (y^2 = x)$ $\quad U_x = U_y = \mathbb{R}$

$\forall_x \forall_y P(x,y)$, $\quad \forall_y \forall_x P(x,y)$

$\boxed{\forall_x \exists_y P(x,y)}$, $\quad \exists_y \forall_x P(x,y)$

$\exists_x \forall_y P(x,y)$, $\quad \forall_y \exists_x P(x,y)$

$\exists_x \exists_y P(x,y)$, $\quad \exists_y \exists_x P(x,y)$

$\forall x \, \exists y \, P(x,y)$ = 'for all real x, there exists a real y such that $y^2 = x$'

= 'every real number has a real square root'

<u>Exercise</u> Translate the other 7 statements, and determine their truth values.

<u>Ex.</u> $R(x,y,z) = (x \cdot y = z)$

$$U_x = U_y = U_z = \mathbb{Q}$$

$$\forall x \forall y \exists z : \underbrace{R(x,y,z)}_{x \cdot y = z}$$

'for any rational x, and any rational y, there exists a rational z such that $z = x \cdot y$'

'the product of any two rational numbers is a rational number'.

Ex. Translate:

'every real number has a real cube root'.

$\forall x \in \mathbb{R} \; \exists y \in \mathbb{R} \; : \; y^3 = x$ ← true

note that order of Quantifiers matters.

$\exists y \in \mathbb{R}\ \forall x \in \mathbb{R} : y^3 = x$ ← false!!

translates as:

'there is a real number that is the cube root of every real number.'

next time: 2.8

2.9

⋮