CSE 16 6-29-21

Chap. 2 Logic

(2.1) statement

<u>Defn</u>

A <u>statement</u> (or <u>proposition</u>) is a sentence that can be evaluated (in principle) as either <u>true</u> or <u>false</u>.

<u>Ex.</u> statements

- 'it is raining today'
- $2+3=5$
- $2+3=7$

Ex. not statements

- 'hello'
- $x+7=2$
- $x+y=z$

open sentences
propositional functions
predicates

notation

- Propositional variables: $P, Q, R, \ldots$

  $p, q, r, \ldots$

- Propositional functions: $P(x)$, $Q(a,b)$, $R(u,v)$

Rmk.

a statement has a truth value, but we might not know what it is.

Ex. Goldbach Conjecture.

Every even integer greater than 2 can be expressed as the sum of two prime numbers.

(2.2) operations: and, or, not, exor

and (conjunction)

notation: $P \wedge Q$

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

True only in case both operations are true.

Ex. P = 'it is raining today'
Q = 'today is wednesday'

or (disjunction, inclusive or)

notation: $P \vee Q$

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

False only in the case both operands are false.

meaning: 'either P is true, or Q is true, or possibly both are true'

Prev ex: P = 'it is raining'
Q = 'today is wednesday'

exor (exclusive or)

notation: $P \oplus Q$

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

True only if both operands have different truth values.

$P \oplus Q$ means: 'either P, or Q is true, but not both'.

Ex. 'your money *or* your life'

not (negation)

notation: $\sim P$ (other texts: $\neg P$)

P	$\sim P$
F	T
T	F

reverse truth value of operand

$\sim P$ means 'not P' or

'it is not the case that P'.

(2.3) operation: conditional

conditional (implication)

notation: $P \Rightarrow Q$ (other: $P \to Q$)

Read: 'if P, then Q'

'if P is true, then Q is also true'

hypothesis $\qquad$ conclusion

P	Q	$P \Rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

False only in the case that hypothesis is true and conclusion false.

Ex. P = 'it is sunny today'

Q = 'I will take you to the beach'

Read $P \Rightarrow Q$ as:

'if P, then Q' $\qquad$ 'Q, if P'

'P implies Q' $\qquad$ 'Q whenever P'

'P only if Q' $\qquad$ 'Q is necessary for P'

'P is sufficient for Q'.

Propositions Related to $P \Rightarrow Q$

- converse : $Q \Rightarrow P$
- contrapositive : $\sim Q \Rightarrow \sim P$
- inverse : $\sim P \Rightarrow \sim Q$

<u>note</u> : $\sim$ has higher precedence than any other logical op.

<u>Ex</u> $\sim P \Rightarrow Q$ means $(\sim P) \Rightarrow Q$, it does not mean $\sim (P \Rightarrow Q)$

(2.4) operation: biconditional

biconditional (iff, if and only if)

notation $P \Leftrightarrow Q$ (also $P \leftrightarrow Q$)

P	Q	$P \Leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

True only when both operands have same truth value.

Read $P \Leftrightarrow Q$ as:

'P if and only if Q'

'P is equivalent to Q'

'P is necessary and sufficient for Q'

(2.5) Truth tables

- Atomic statements: $P, Q, R, \ldots$
- Compound statements:
  statements built from atomic stmts. using logical operators.

$$P \Rightarrow Q$$
$$\sim P \Rightarrow (R \vee S)$$
$$(P \wedge Q) \Rightarrow (\sim R \vee \sim Q)$$

note: # rows in a truth table for a compound prop. is

$$2^{(\#\text{ of variables})}$$

Ex $(P \Leftrightarrow Q) \vee (\sim Q \Rightarrow R)$

P	Q	R	$P \Leftrightarrow Q$	$\sim Q$	$\sim Q \Rightarrow R$	$(P \Leftrightarrow Q) \vee (\sim Q \Rightarrow R)$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

Ex $\sim(P \vee Q)$

P	Q	$P \vee Q$	$\sim(P \vee Q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

same

Ex $\sim P \wedge \sim Q$

P	Q	$\sim P$	$\sim Q$	$(\sim P) \wedge (\sim Q)$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

evidently: $\sim(P \vee Q) = \sim P \wedge \sim Q$

Know: $P \Rightarrow Q \neq Q \Rightarrow P$

what about : $P \Rightarrow (Q \Rightarrow R) \stackrel{?}{=} (P \Rightarrow Q) \Rightarrow R$

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow R$
0	0	0	1	1	1	0
0	0	1	1	1	1	1
0	1	0	0	1	1	0
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	0	0	1	0
1	1	1	1	1	1	1

different!!

$\therefore P \Rightarrow (Q \Rightarrow R) \neq (P \Rightarrow Q) \Rightarrow R$

$\therefore P \Rightarrow Q \Rightarrow R$ is meaningless

Exercise

show that $P \vee Q \wedge R$ is meaningless by constructing truth tables for $(P \vee Q) \wedge R$ and $P \vee (Q \wedge R)$.

(2.6) Logical Equivalence

Defn

A tautology is a (compound) Proposition that is true for any assignment of truth values to its Propositional variables.

Ex. $P \vee \sim P$