CSE 16 6-22-21

Why Prove things in Engineering?

- clear thinking
- Know <u>why</u> things are true.
- ultimate explanation.
- Proofs are formal essays, similar to a computer Program.

Chapter 1 Sets

(Naïve) Definition

A set is any (unordered) collection of objects, of any kind.

Note: this defn. leads to Paradoxes!

Objects in a set are called members or elements.

Write: $x \in S$

$\in$ "is an element of"

To specify a set

<u>Ex.</u> ~~to~~ list elements in braces $\{\ \}$

$\{1, 2, 3\}$

$\{1, 2, 3, \ldots, 99, 100\}$

$\{1, 2, 3, \ldots\}$

Important sets!

- natural numbers:

  $\mathbb{N} = \{0, 1, 2, \ldots\}$ use this

  <u>note</u>: book says $\mathbb{N} = \{1, 2, 3, \ldots\}$

- Integers:

  $\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

  $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

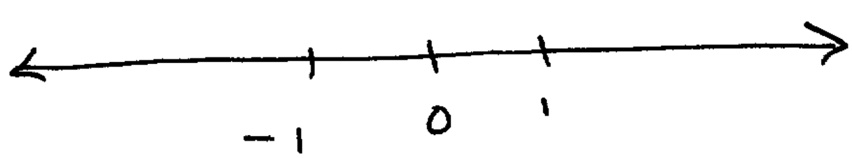
  $\mathbb{Z}^- = \{\ldots -3, -2, -1\}$

- Rational numbers

$$\mathbb{Q} = \{ \text{---- fractions } \tfrac{p}{q} \text{ ----} \}$$

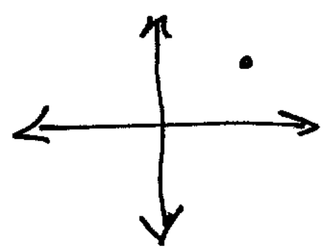
- Real numbers

$\mathbb{R}$



- Complex numbers

$\mathbb{C}$



- Empty set

$$\emptyset = \{ \ \}$$

the set with no elements.

note:

two sets are equal iff they contain the same elements.
order and repetition is irrelevant.

$$\{1,2,3\} = \{2,3,1\}$$
$$= \{1,2,1,1,1,3,2,3,3,2,2,1\}$$

so if A, B are sets, then

$$A = B$$

iff both

- every element of A belongs to B

and - " " " B " " A

set builder notation

can specify a set as

$$A = \{ \underline{\text{expression}} \mid \underline{\text{property}} \}$$

↑

"such that"

A is the set of all values of <u>expression</u> that satisfy <u>property</u>

<u>Ex.</u> $\{ x \mid x \text{ is a prime number} \}$

$$= \{ 2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots \}$$

<u>Ex.</u> same thing

$$\{ n \in \mathbb{N} \mid n \text{ is prime} \}$$

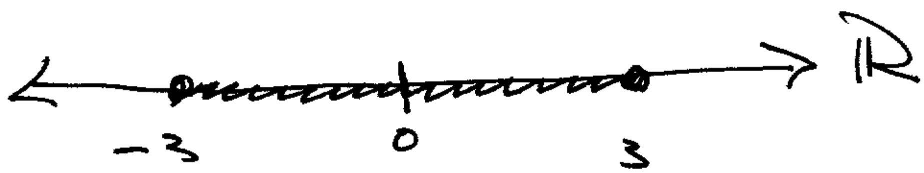
Ex. $\{n \in \mathbb{Z} \mid |n| \leq 3\}$

$= \{-3, -2, -1, 0, 1, 2, 3\}$

Ex. $\{x \in \mathbb{R} \mid |x| \leq 3\}$

$= [-3, 3]$

Picture:



Ex.

$\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$

Defn

A set S is called finite if it contains exactly n elements for some $n \in \mathbb{N}$. If S is not finite, it's called infinite.

notation

- $|S| = n$: S is finite with n elements.
- $|S| < \infty$: S is finite
- $|S| = \infty$: S is infinite

$|S|$ is called the cardinality of set S.

(1.2) Cartesian Products

Defn
an ordered collection of n objects (possibly repeated) is called an ordered n-tuple. (also called a finite sequence)

notation $(x_1, x_2, x_3, \ldots, x_n)$

- If $n=2$: ordered pair
- If $n=3$: ordered triple

note: $(x_1, x_2) = (y_1, y_2)$

iff both $x_1 = y_1$ and $x_2 = y_2$.

<u>Def'n</u>

The <u>Cartesian Product</u> of two sets A, B is the set of all ordered pairs (x, y) where $x \in A$ and $y \in B$. <u>notation</u>: $A \times B$

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

<u>Ex.</u> $A = \{1, 2\}$, $B = \{3, 4, 5\}$

$$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$$

$$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$$

$B \times B = \ldots$ exercise ...

$B \times A = \ldots$ exercise ...

Theorem

If A, B are finite sets, then so is $A \times B$, and

$$|A \times B| = |A| \cdot |B|$$

more generally, the cartesian product of n sets $A_1, A_2, A_3, \ldots, A_n$ is

$$A_1 \times A_2 \times A_3 \times \cdots \times A_n$$

$$= \{(x_1, x_2, x_3, \ldots, x_n) \mid x_i \in A_i \text{ for } 1 \le i \le n\}$$

Thm if sets A_i are finite, then

$$|A_1 \times A_2 \times A_3 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot |A_3| \cdots |A_n|$$