

CSE 16
Summer 2020
Quiz 3

Solutions

1. (20 Points) In a group of 35 people, all of whom drink coffee or tea (or both), 18 drink coffee and 12 drink both coffee and tea.
- a. (10 Points) How many in the group drink tea?

Solution:

Let $A = \{\text{coffee drinkers}\}$ and $B = \{\text{tea drinkers}\}$. By the principle of inclusion-exclusion, we have $|A \cup B| = |A| + |B| - |A \cap B|$. The given information says $|A \cup B| = 35$, $|A| = 18$ and $|A \cap B| = 12$. Therefore $35 = 18 + |B| - 12$, whence $|B| = 29$. Thus **29 people drink tea.** ■

- b. (10 Points) How many drink only tea (i.e. tea and not coffee)?

Solution:

Using the same notation as in (a), we have $|B - A| = |B - (A \cap B)| = 29 - 12 = 17$. Therefore **17 people drink only tea.** ■

2. (15 Points) Determine the coefficient of x^{-6} in the full expansion of $\left(x + \frac{1}{x}\right)^{10}$.

Solution:

The Binomial Theorem gives

$$\left(x + \frac{1}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \cdot x^{10-k} \cdot \left(\frac{1}{x}\right)^k = \sum_{k=0}^{10} \binom{10}{k} \cdot x^{10-2k}$$

Setting $10 - 2k = -6$, we obtain $2k = 16 \Rightarrow k = 8$. Therefore the coefficient of x^{-6} is

$$\binom{10}{8} = \frac{10 \cdot 9}{2} = 5 \cdot 9 = \mathbf{45}$$

■

3. (15 Points) Let A be a finite set with exactly 35 subsets of size 4. Determine $|A|$.

Solution:

Let $n = |A|$. We must solve $\binom{n}{4} = 35$ for n . Examining Pascale's triangle down to row 7, we find that $\binom{7}{4} = 35$, whence $|A| = \mathbf{n = 7}$. ■