CSE 16
Summer 2020
Quiz 2 **Solutions**

1. (15 Points) Let $K(x, y)$ be the statement "x knows y", where the domain for both x and y is the set of all people. (Assume it is possible for x to know y while y does not know x, and hence $K(x, y)$ and $K(y, x)$ are different statements.) Translate "there is a person who is known by everyone" into Symbolic Logic.

   Choose one answer below.

   a. $\exists x \sim\forall y \;\; K(x, y)$

   b. $\forall x\, \exists y \;\; K(x, y)$

   c. $\sim\exists y\, \forall x \;\; K(y, x)$

   d. $\exists x\, \forall y \;\; K(x, y)$

   e. $\exists y\, \forall x \;\; K(x, y)$

   f. $\forall y\, \exists x \;\; K(x, y)$

   Solution: **(e)** $\exists \boldsymbol{y}\, \forall \boldsymbol{x} \;\; \boldsymbol{K}(\boldsymbol{x}, \boldsymbol{y})$

2. (20 Points) Let the universe for the following quantified statements be $\mathbb{N}$. Define propositional functions $P(n)$ and $Q(n)$ to be

$$P(n) = (\exists k : n = 3k)$$

   and

$$Q(n) = (\exists k : n = 5k + 2)$$

   Determine the 3 *smallest* elements in the set

$$\{\, n \in \mathbb{N} \mid P(n) \wedge Q(n) \,\}$$

   Solution:
   Let $A = \{\, n \in \mathbb{N} \mid P(n)\}$ and $B = \{\, n \in \mathbb{N} \mid Q(n) \,\}$. Then

$$A = \{\, 3, 6, 9, \mathbf{12}, 15, 18, 21, 24, \mathbf{27}, 30, 33, 36, 39, \mathbf{42}, 45, 48, \dots \dots \dots \}$$

   and

$$B = \{\, 7, \mathbf{12}, 17, 22, \mathbf{27}, 32, 37, \mathbf{42}, 47, 52, 57, \dots \dots \dots \}.$$

   Thus $\{\, n \in \mathbb{N} \mid P(n) \wedge Q(n) \,\} = A \cap B = \{\, 12, 27, 42, \dots \dots \}$, whose 3 smallest elements are **12, 27, 42**.

3. (15 Points) A banking system requires Personal Identification Numbers (PINs) that satisfy two constraints.

    (1) Must consist of only decimal digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and
    (2) Must have length at least 5 and at most 8.

    How many different PINs are there?

    Solution:
    We partition the acceptable PINs into 4 sets.

    length 5: # of PINs $= 10^5$
    length 6: # of PINs $= 10^6$
    length 7: # of PINs $= 10^7$
    length 8: # of PINs $= 10^8$

    By the addition principle, the number of valid PINs is $10^5 + 10^6 + 10^7 + 10^8 = \mathbf{111{,}100{,}000}$.