CSE 16
Homework Assignment 3
Solutions to Selected Problems

(3.3) 12

Six math books, four physics books and three chemistry books are arranged on a shelf. How many arrangements are possible if all books of the same subject are grouped together?

Solution:

We arrange the books in four steps. First arrange the subjects, then arrange the books in each subject.

(1) 3! ways to arrange the 3 subjects
(2) 6! ways to arrange the 6 math books
(3) 4! ways to arrange the 4 physics books
(4) 3! ways to arrange the 3 chemistry books

By the multiplication principle, there are $\mathbf{3! \cdot 6! \cdot 4! \cdot 3! = 622,080}$ ways to arrange the books. ■

(3.4) 10

How many permutations of the digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are there in which the digits alternate even and odd? (For example 2183470965.)

Solution:

We first choose a permutation of the even digits $\{0, 2, 4, 6, 8\}$, then choose a permutation of the odd digits $\{1, 3, 5, 7, 9\}$. The number of such arrangements is then $\mathbf{5! \cdot 5! = 14,400}$, by the multiplication principle.

■

(3.5) 4

Suppose the set B has the property that $|\{X \mid X \in \mathbb{P}(B) \text{ and } |X| = 6\}| = 28$. Determine $|B|$.

Solution:

Let $|B| = n$. It is required that B have exactly 28 subsets of cardinality 6, so we must solve the equation

$$\binom{n}{6} = 28$$

for n. Using the formula for binomial coefficients, we need to find the roots of a 6^{th} degree polynomial. It's much easier to just examine Pascal's triangle until we find the number 28 in the 6^{th} column. We find that this occurs in row 8:

$$\binom{8}{6} = 28,$$

and hence $\boldsymbol{|B| = 8}$. ■

<u>(3.6) 6</u>

Show that $\sum_{k=0}^{n}\binom{n}{k} = 2^n$.

Solution:

The wording of this question says to use definition 3.2 (page 85) and fact 1.3 (page 13). If you look at what's on those pages, they're really asking for a combinatorial proof. We will accept either a combinatorial or algebraic proof, both of which are presented below.

<u>Combinatorial Proof:</u>

Both sides count the number of subsets of an n-element set. The right hand side, 2^n, does so by fact 1.3 on page 13. The left hand side does so since, by definition 3.2 on page 85, $\binom{n}{k}$ denotes the number of k-element subsets of an n-element set. Summing this quantity for $0 \leq k \leq n$ gives the *total* number of subsets of an n-element set, and hence LHS $=$ RHS. ■

<u>Algebraic Proof:</u>

The formula follows instantly by the Binomial Theorem upon setting $x = y = 1$. In this case

$$2^n = (1+1)^n = \sum_{k=0}^{n}\binom{n}{k}\cdot 1^{n-k}\cdot 1^k = \sum_{k=0}^{n}\binom{n}{k}$$

■