CSE 16
Homework Assignment 2
Solutions to Selected Problems

(2.5) 10
Suppose the statement $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (Note this can be done without constructing a truth table.)

Solution:
The only way for an implication to be false is for the hypothesis to be true while the conclusion is false. Thus $(P \wedge Q) \vee R$ is true, and $R \vee S$ is false. Since $R \vee S$ is false, both operands must be false, i.e.

$$R = S = F.$$

Since R is false and $(P \wedge Q) \vee R$ is true, we must have $P \wedge Q$ true, hence both operands are true, i.e.

$$P = Q = T.$$

Thus if $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false, then $\boldsymbol{P = Q = T}$**, and** $\boldsymbol{R = S = F}$. ∎

(2.6) 6
Show that $\sim (P \wedge Q \wedge R) = (\sim P) \vee (\sim Q) \vee (\sim R)$ using truth tables.

Solution:

P	Q	R	$P \wedge Q \wedge R$	$\sim (P \wedge Q \wedge R)$	$\sim P$	$\sim Q$	$\sim R$	$(\sim P) \vee (\sim Q) \vee (\sim R)$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

Since LHS and RHS have the same truth table, **they are logically equivalent**. ∎

<u>(2.6)   10</u>
Decide whether $(P \Rightarrow Q) \vee R$ and $\sim \big((P \wedge \sim Q) \wedge \sim R\big)$ are logically equivalent.

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$\sim Q$	$P \wedge \sim Q$	$\sim R$	$(P \wedge \sim Q) \wedge \sim R$	$\sim \big((P \wedge \sim Q) \wedge \sim R\big)$
0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	0	0	1
0	1	0	1	1	0	0	1	0	1
0	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	1	0	0	1
1	1	0	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0	0	1

Since the two expressions have the same truth table, **they are logically equivalent**. ■

(2.7)   4
Translate $\forall\, X \in \mathbb{P}(\mathbb{N}),\ X \subseteq \mathbb{R}$ into natural language, and determine whether it is true or false.

Solution:
Any one of the following translations will work.

1st translation: "for every subset *X* of the positive integers, *X* is a subset of the real numbers"
2nd translation: "every subset of the positive integers is a subset of the real numbers"
3rd translation: "every positive integer is a real number"

Note that a simpler way to say the same thing in symbols would be: $\mathbb{N} \subseteq \mathbb{R}$. ■