CSE 16 6-30-20

Chapter 2 : Logic

(1.1) statements

Defn

a statement (or proposition) is a sentence that can be assigned a truth value : true or false (at least in principle.)

statements :

- 'It is raining today'
- $2+3=5$
- $2+3=7$

not statements

- 'Hello'
- $x + 7 = 2$
- $a + b = c$

truth value depends on values of variables.

last 2 items are called <u>open sentences</u> (also <u>propositional functions</u>, or <u>predicates</u>).

Propositional variables : $P, Q, R, \ldots$

Propositional functions : $P(x), Q(y), R(a, b, c), \ldots$

<u>note:</u>

we might not actually know the truth value of a statement.

<u>EX</u>. Goldbach Conjecture

Every even integer greater than 2 is the sum of 2 primes.

$4 = 2 + 2$

$6 = 3 + 3$

$8 = 3 + 5$

$10 = 5 + 5$

$\vdots$

(2.2) logical operations: and, or, not

and (conjunction)

notaion: $P \wedge Q$

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

True only in the case when both operands are true.

Ex. P = 'it is raining today'
Q = 'today is Wednesday'

$P \wedge Q$ = 'it is raining today and today is Wednesday'.

or (disjunction, inclusive or)

notation : $P \vee Q$

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

False only in case both operands are false

$P \vee Q$ means : 'either P, or Q, or possibly both, are true'.

exor (exclusive or)

notation : $P \oplus Q$

means : 'either P, or Q, but not both are true'

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

True only when both operands are different truth values.

<u>Ex</u> 'your money or your life.'

<u>not (negation)</u>

notation : $\sim P$

P	$\sim P$
F	T
T	F

revers truth value of the operand.

read $\sim P$ as 'not P'

(2.3) conditional operator

conditional (implication)

notation: $P \Rightarrow Q$ (also $P \to Q$, $P \supset Q$)

P	Q	$P \Rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

False only in case hypothesis true and conclusion false.

left operand: <u>hypothesis</u>

right operand: <u>conclusion</u>

<u>read $P \Rightarrow Q$ as:</u>

'if P, then Q'

'P implies Q'

'P only if Q'

'P is sufficient for Q'

'Q if P'

'Q whenever P'

'Q is necessary for P'

consider example:

P = 'it is sunny today'

Q = 'I will take you to the beach'

$P \Rightarrow Q$ = 'if it is sunny today, then I will take you to the beach'.

statements related to $P \Rightarrow Q$:

- converse: $Q \Rightarrow P$
- contrapositive: $\sim Q \Rightarrow \sim P$
- inverse: $\sim P \Rightarrow \sim Q$

<u>note</u>: $\sim$ has higher precedence than all other operators, so

$\sim Q \Rightarrow \sim P$ means $(\sim Q) \Rightarrow (\sim P)$

$\sim P \vee Q$ means $(\sim P) \vee Q$

(2.4) biconditional operator

<u>biconditional</u>

notation: $P \Leftrightarrow Q$ (also $P \leftrightarrow Q$)

P	Q	$P \Leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

True only when operands have same truth value

Read as:

'P if and only if Q' ('P iff Q')

'P is necessary and sufficient for Q'

'P is equivalent to Q'

(2.5) Truth tables

- atomic statements: $P, Q, R, \ldots$
- compound statements: built up from prop. variables using logical operators

Ex. $(P \Leftrightarrow Q) \vee (\sim Q \Rightarrow R)$

P	Q	R	$P \Leftrightarrow Q$	$\sim Q$	$\sim Q \Rightarrow R$	$(P \Leftrightarrow Q) \vee (\sim Q \Rightarrow R)$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

(1 = T = true, 0 = F = false)

Ex. $\sim(P \vee Q)$ vs. $(\sim P) \vee Q$

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$(\sim P) \vee Q$
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	0	0	0
1	1	1	0	0	1

different

Ex. $P \Rightarrow (Q \Rightarrow R)$ ①

vs.

$(P \Rightarrow Q) \Rightarrow R$ ②

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	1	2
0	0	0	1	1	1	0
0	0	1	1	1	1	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

different

so

$$\underbrace{P \Rightarrow (Q \Rightarrow R)}_{1} \neq \underbrace{(P \Rightarrow Q) \Rightarrow R}_{2}$$

never write!

$$P \Rightarrow Q \Rightarrow R$$

Exercise

compare

$$(P \vee Q) \wedge R$$

vs.

$$P \vee (Q \wedge R)$$

using truth table analysis.

answer: different

so cannot write

$$P \vee Q \wedge R$$

next: logical equivalence