CSE 16 6-24-20

- Piazza: self signup
  required reading
- lecture recording
- add links

last time ....

<u>Thm</u> if S is finite, so is $\mathbb{P}(S)$ and

$$|\mathbb{P}(S)| = 2^{|S|}$$

Ex.

$\mathbb{P}(\phi) = \{\phi\}$ note: $\boxed{2^0 = 1}$

note:

$\{\phi\} = \{\{\ \}\} \neq \{\ \} = \phi$

$$\mathbb{P}(\mathbb{P}(\phi)) = \mathbb{P}(\{\phi\})$$
$$= \{\phi, \{\phi\}\} \qquad \boxed{2^1 = 2}$$

$$\mathbb{P}(\mathbb{P}(\mathbb{P}(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\} \qquad \boxed{2^2 = 4}$$

$$|\mathbb{P}(\mathbb{P}(\mathbb{P}(\mathbb{P}(\phi))))| = 2^4 = 16$$

Exercise:

find an expression for

$$\left|\underbrace{\mathbb{P}(\mathbb{P}(\mathbb{P}(\cdots\cdots\mathbb{P}}_{n=\#\,\mathbb{P}\text{'s}}(\emptyset)\cdots)))\right|$$

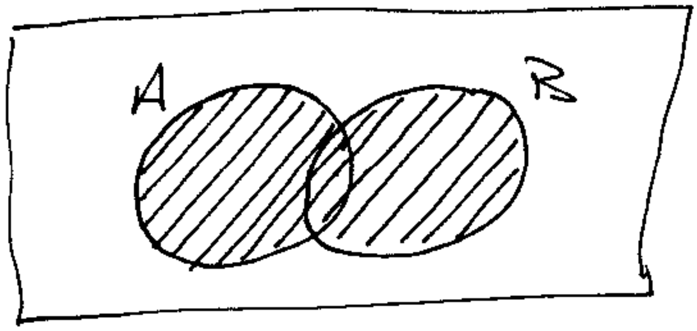
$n=0$: 2^0
$n=1$: 2^{2^0}
$n=2$: $2^{2^{2^0}}$

(1.5) set operations

let A, B be sets

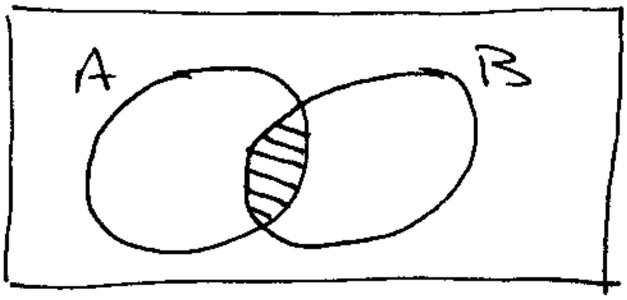
- union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Venn diagram



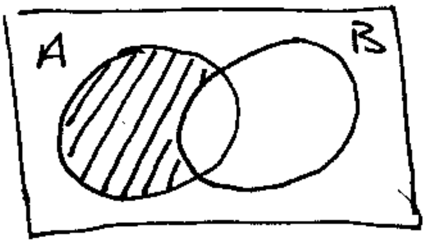
- intesection

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

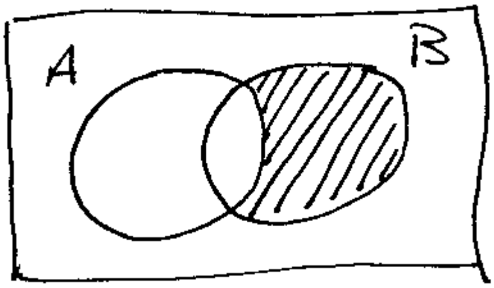


- difference

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$



observe for any A, B

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

but

$$A - B \neq B - A$$

unless $A = B$, in which case $A - B = \emptyset$.

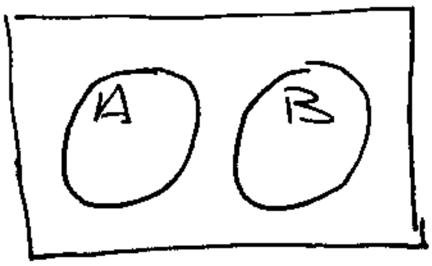
<u>Ex.</u> $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$

$$A \cup B = \{1, 2, 3, 4, 5\}$$
$$A \cap B = \{3, 4\}$$
$$A - B = \{1, 2\}$$
$$B - A = \{5\}$$
$$A - A = \emptyset = B - B$$

Defn

we say A, B are disjoint iff

$A \cap B = \phi$



observe for any A, B we have

$A - B$ and B are disjoint

$$(A-B) \cap B = \phi$$

note: $A - B \subseteq A$

$B - A \subseteq B$

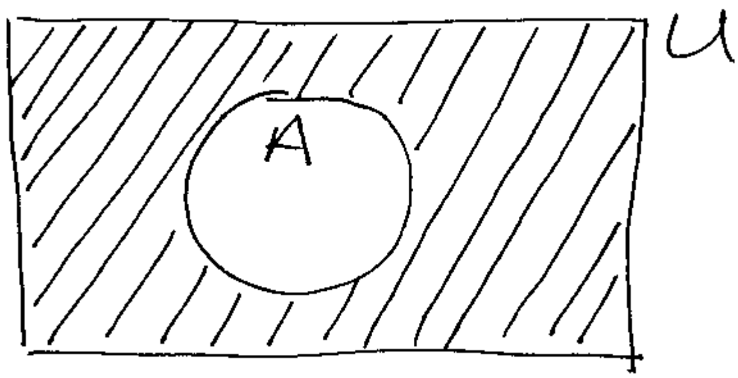
also $(A-B) \cap (B-A) = \phi$

(1.6) complement

suppose all sets under discussion are subsets of some universal set U.

Defn
The complement of A (relative to U) is

$$\bar{A} = U - A$$
$$= \{x \mid x \notin A\}$$
$$= \{x \in U \mid x \notin A\}$$



Ex. let $U = \mathbb{N}$ and

$$A = \{x \in \mathbb{N} \mid x \geq 8\} = \{8, 9, 10, \ldots\}$$

then

$$\bar{A} = \{x \in \mathbb{N} \mid x \notin A\}$$

$$= \{x \in \mathbb{N} \mid x < 8\}$$

$$= \{1, 2, 3, \ldots, 7\}$$

Ex. let $U = \mathbb{Z}$

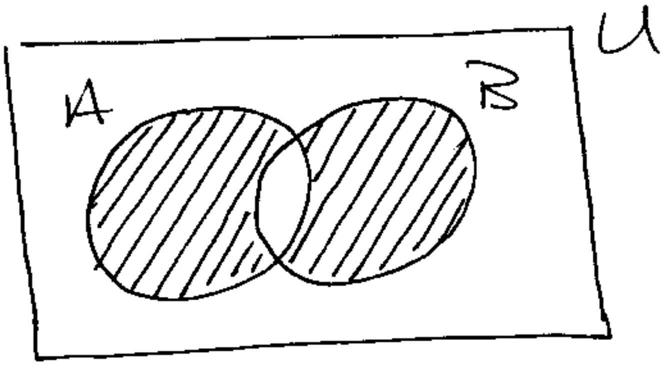
$$A = \{x \in \mathbb{Z} \mid x \geq 8\} = \{8, 9, 10, \ldots\}$$

$$\bar{A} = \{\ldots -2, -1, 0, 1, 2, \ldots, 7\}$$

(1.7) Venn Diagrams

new operation: symmetric difference

$$A \oplus B = (A - B) \cup (B - A)$$



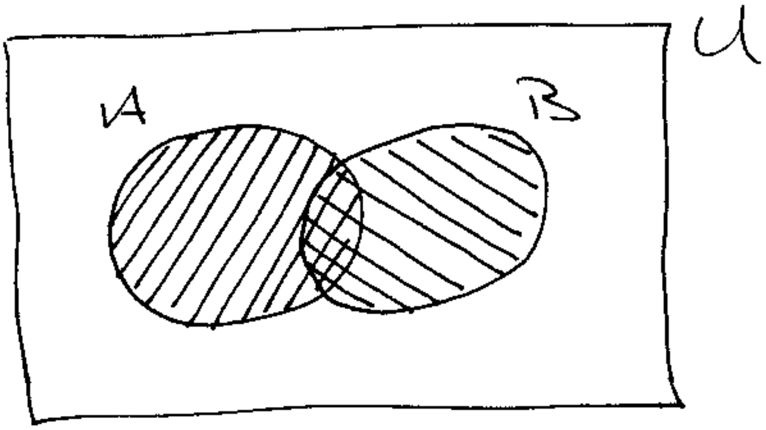
observe:

$$A \oplus B = (A \cup B) - (A \cap B)$$

Theorem

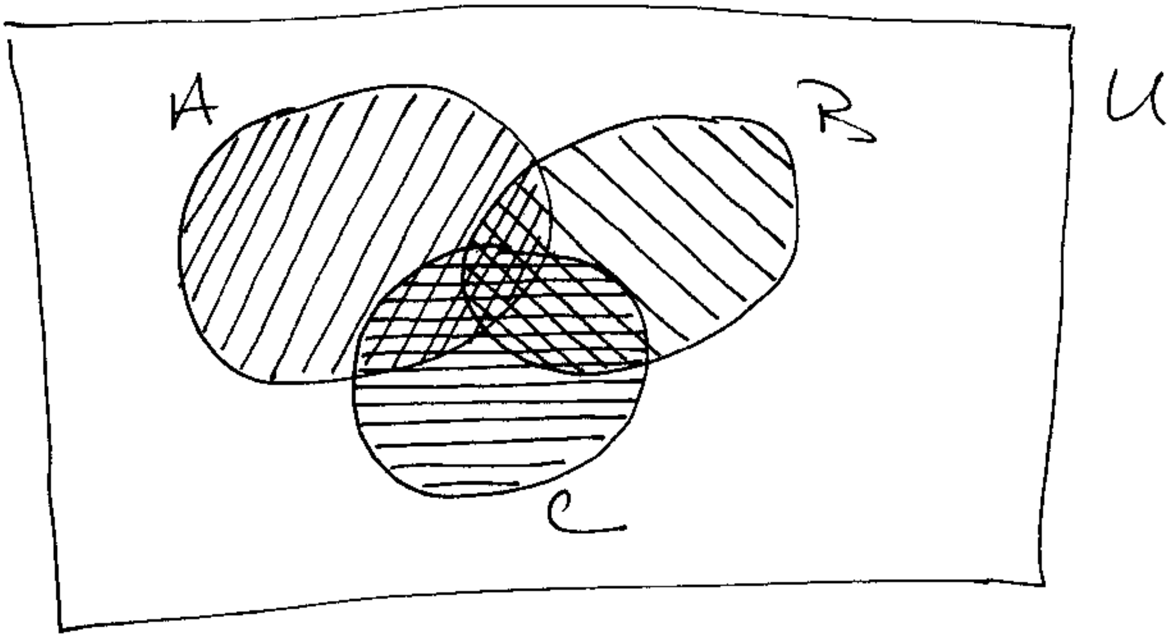
If A, B are finite sets, so are $A \cup B$ and $A \cap B$, and

$$|A \cup B| = |A| + |B| - |A \cap B|$$



This a special case of the Principle of Inclusion-Exclusion (PIE).

3-set version



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

<u>Exercise</u>:
- try to write the 4 set version of PIE.
- try to write n set version.

(1.8) Indexed Sets

Let $A_1, A_2, A_3, \ldots\ldots\ldots$ be a list (possibly infinite) of sets.

notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$$

$$= \{x \mid x \in A_i \text{ for } \underbrace{\text{at least one}}_{\underline{\text{some}}} \ 1 \le i \le n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots$$

$$= \{x \mid x \in A_i \text{ for } \underline{\text{some}} \ i \ge 1\}$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots\cdots A_n$$

$$= \{x \mid x \in A_i \text{ for } \underline{\text{all}}\ 1 \le i \le n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots\cdots\cdots$$

$$= \{x \mid x \in A_i \text{ for } \underline{\text{all}}\ i \ge 1\}$$

<u>Ex.</u>

$$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7\}$$

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \{4\}$$

let $A_1 = \{1, 2, 3, 4\}$

$A_2 = \{2, 4, 5\}$

$A_3 = \{3, 4, 6, 7\}$

can also pick out specific indices $i \in \mathbb{N}$.

$$\bigcup_{i \in \{2,7,11\}} A_i = A_2 \cup A_7 \cup A_{11}$$

$$\bigcap_{i \in \{2,7,11\}} A_i = A_2 \cap A_7 \cap A_{11}$$

In general suppose we have a set A_α for every $\alpha \in I$, where I is any set (finite or infinite), called the index set. Then

$$\bigcup_{\alpha \in I} A_\alpha = \{x \mid x \in A_\alpha \text{ for } \underline{\text{some}}\ \alpha \in I\}$$

$$\bigcap_{\alpha \in I} A_\alpha = \{x \mid x \in A_\alpha \text{ for } \underline{\text{all}}\ \alpha \in I\}$$

Ex suppose $J \neq \phi$ and $J \subseteq I$.
does it follow that

$$(*) \qquad \bigcup_{\alpha \in J} A_\alpha \subseteq \bigcup_{\alpha \in I} A_\alpha$$

more concrete example. Let $J = \{1, 2\}$, $I = \{1, 2, 3\}$. Then $(*)$ becomes

$$A_1 \cup A_2 \subseteq A_1 \cup A_2 \cup A_3$$

This is true.