CSE 16 6-23-20

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why do we prove things in an Engineering class?

- ultimate explanation.
- proofs are (a little) like programs.
- clear thinking

chap 1 : Set Theory

<u>Defn</u> (Naive)
a <u>set</u> is any (unordered) collection of objects of any kind.

leads directly to Paradoxes!!

The objects belonging to a set are called <u>members</u> or <u>elements</u>.

write $x \in S$

to mean x is an element of set S.

can specify a set by listing its members in braces $\{ \}$

<u>Ex</u> $\{1, 2, 3\}$

$= \{3, 1, 2\} = \{2, 3, 1\} = \cdots$

<u>Ex.</u> $\{1, 2, 3, \ldots\ldots, 100\}$

<u>Ex.</u> $\{1, 2, 3, \ldots\ldots\ldots\}$

repetitions don't matter

<u>Ex</u> $\{1, 2, 3\} = \{2, 1, 1, 1, 3, 3, 2, 3, 1\}$

Some important sets

- natural numbers

$$\mathbb{N} = \{1, 2, 3, \ldots\ldots\}$$

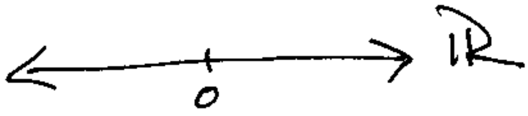
also called positive integers.

- integers

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

$$\mathbb{Z}^+ = \{1, 2, 3, \ldots\} = \mathbb{N}$$

$$\mathbb{Z}^- = \{\ldots, -3, -2, -1\}$$

- Rational numbers : $\mathbb{Q}$
- Real numbers : $\mathbb{R}$ 

- Complex numbers : $\mathbb{C}$

- Empty set

$$\phi = \{ \}$$

the set with no members

Note: two sets are considered equal iff they have same members.

$$A = B$$

Set builder notation

$$A = \{ \text{expression} \mid \text{property} \}$$

A is the set of all values of expression that satisfy property

Ex. $\{x \mid x \text{ is a prime number}\}$

$= \{2, 3, 5, 7, 11, 13, 17, 23, \ldots\}$

Ex. same set

$\{n \in \mathbb{N} \mid n \text{ is prime}\}$

Ex. $\{n \in \mathbb{Z} \mid |n| \leq 3\}$

$= \{-3, -2, -1, 0, 1, 2, 3\}$

Ex. $\{x \in \mathbb{R} \mid |x| \leq 3\}$

$= [-3, 3]$

Graph:

Ex

$$\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$$

Defn

A set is called finite iff it contains exactly n elements, for some $n \in \mathbb{Z}$, $n \geq 0$. If S is not finite, its called infinite.

notation: $|S| = n$ 'S is finite, n elements'

write: $|S| < \infty$ 'S is finite'

write: $|S| = \infty$ 'S is infinite'

$|S|$ is called the cardinality of S.

(1.2) Cartesian Products

an ordered collection of n objects is called an <u>ordered n-tuple</u>.

notation: $(x_1, x_2, \ldots, x_n)$

If $n=2$, call this an <u>ordered pair</u>. If $n=3$, an <u>ordered triple</u>

<u>note</u>: $(x_1, x_2) = (y_1, y_2)$ iff both $x_1 = y_1$ and $x_2 = y_2$.

<u>Defn</u> the <u>Cartesian Product</u> of A, B is the set

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

Ex $A=\{1,2\}$, $B=\{3,4,5\}$

$$A\times B=\{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$$

$$B\times A=\{(3,1),(3,2),(4,1),(4,2),(5,1),(5,2)\}$$

observe: $A\times B\neq B\times A$ (in general)

Theorem

if A,B are finite sets, then $A\times B$ is also finite, and

$$|A\times B|=|A|\cdot|B|$$

more generally the Cartesian Product of $A_1, A_2, \ldots, A_n$ is

$$A_1\times A_2\times\cdots\times A_n$$
$$=\{(x_1,x_2,\ldots,x_n)\mid x_i\in A_i \text{ for } 1\le i\le n\}$$

Thm

if $A_1, \ldots, A_n$ are finite

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdots |A_n|$$

Ex. $A = \{1, 2\}$

$B = \{3, 4, 5\}$

$C = \{2, 3\}$

$$A \times B \times C = \{(1,3,2), (1,3,3), (1,4,2), (1,4,3), \text{ - - - - - - - - - - - - - - } \}$$

8 more

exercise

(1.3) subsets

Defn
we say A is a <u>subset</u> of B iff every member of A is also a member of B

notation: $A \subseteq B$

we say A is a proper subset of B iff $A \subseteq B$ and $A \neq B$

notation: $A \subsetneq B$

observe for any set S:

$$S \subseteq S$$

and

$$\phi \subseteq S$$

Remark

some texts use $\subset$ for proper subset, and some $\subset$ for subset.

Rmk

'contains', 'in'

$x \in S$ 'S contains x'

$A \subseteq S$ 'S contains A'

(1.4) Power sets

Defn
The set of all subsets of a set S is called the Power set of S.

notation: $\mathbb{P}(S)$

<u>Ex</u> $S = \{1, 2, 3\}$ $|S| = 3$

subsets of S: #sets

$$\mathbb{P}(S) = \{ \phi \qquad 1$$

$$\{1\}, \{2\}, \{3\}, \qquad 3$$

$$\{1,2\}, \{1,3\}, \{2,3\}, \qquad 3$$

$$\{1,2,3\} \quad \} \qquad 1$$

observe

$$|\mathbb{P}(S)| = 8 = 2^3 = 2^{|S|}$$

<u>Thm</u>
if S is finite, so is $\mathbb{P}(S)$
and

$$|\mathbb{P}(S)| = 2^{|S|}$$