CSE 16
Spring 2024
Quiz 2 **Solutions**

1. (15 Points) Determine the following power sets.

   a. (5 Points) $\mathbb{P}(\mathbb{P}(\emptyset))$

   Solution: $\mathbb{P}(\mathbb{P}(\emptyset)) = \mathbb{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

   ■

   b. (10 Points) $\mathbb{P}(S)$ where $S = \{1, \{1\}, \{1,2\}\}$

   Solution:

   $$\mathbb{P}(S) = \{\emptyset, \{1\}, \{\{1\}\}, \{\{1,2\}\}, \{1,\{1\}\}, \{1,\{1,2\}\}, \{\{1\},\{1,2\}\}, \{1,\{1\},\{1,2\}\}\}$$

   ■

2. (20 Points) Let $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$. Define $f: A \to B$ by

$$\begin{array}{c} f \\ 1 \to a \\ 2 \to a \\ 3 \to c \\ 4 \to b \end{array}$$

   Answer the following questions regarding f. No justifications are necessary for a-d, but explain why your example in part (e) is correct.

   a. (4 Point) Is f injective? **No**

   b. (4 Point) Is f surjective? **Yes**

   c. (4 Point) Is f bijective? **No**

   d. (4 Point) Determine $f(\{2,3\})$. **Solution:** $f(\{2,3\}) = \{\boldsymbol{a}, \boldsymbol{c}\}$

   e. (4 Points) Find subsets $U, V \subseteq A$ such that $f(U) \cap f(V) \neq f(U \cap V)$.

   Solution: (Note there are many valid solutions to this problem.)
   Let $\boldsymbol{U} = \{\mathbf{1},\mathbf{3}\}$ and $\boldsymbol{V} = \{\mathbf{2},\mathbf{4}\}$. Then $U \cap V = \emptyset$, whence $f(U \cap V) = f(\emptyset) = \emptyset$. On the other hand, $f(U) = \{a,c\}$ and $f(V) = \{a,b\}$, so $f(U) \cap f(V) = \{a\} \neq \emptyset$. ■

   Some Alternate Solutions:
   - $\boldsymbol{U} = \{\mathbf{1},\mathbf{3}\}$ **and** $\boldsymbol{V} = \{\mathbf{2},\mathbf{3}\}$. Then $U \cap V = \{3\}$, so $f(U \cap V) = f(\{3\}) = \{c\}$. However, $f(U) = \{a,c\}$ and $f(V) = \{a,c\}$, so $f(U) \cap f(V) = \{a,c\} \neq \{c\}$.
   - $\boldsymbol{U} = \{\mathbf{1},\mathbf{3},\mathbf{4}\}$ **and** $\boldsymbol{V} = \{\mathbf{2},\mathbf{4}\}$. Then $U \cap V = \{4\}$, so $f(U \cap V) = \{b\}$. But, $f(U) = \{a,b,c\}$ and $f(V) = \{a,b\}$, so $f(U) \cap f(V) = \{a,b\} \neq \{b\}$.

3. (15 Points) Show that the sequence $(1 + 2 \cdot 3^n)_{n=0}^{\infty}$ is a solution to the recurrence relation

$$a_n = 4a_{n-1} - 3a_{n-2} \qquad \text{(for } n \geq 2).$$

Proof:
We substitute $a_n = 1 + 2 \cdot 3^n$ into the right hand side of the above recurrence to get

$$\begin{aligned}
\text{RHS} &= 4a_{n-1} - 3a_{n-2} \\
&= 4(1 + 2 \cdot 3^{n-1}) - 3(1 + 2 \cdot 3^{n-2}) \\
&= (4 - 3) + 8 \cdot 3^{n-1} - 6 \cdot 3^{n-2} \\
&= 1 + 8 \cdot 3^{n-1} - 2 \cdot 3 \cdot 3^{n-2} \\
&= 1 + (8 - 2)3^{n-1} \\
&= 1 + 6 \cdot 3^{n-1} \\
&= 1 + 2 \cdot 3 \cdot 3^{n-1} \\
&= 1 + 2 \cdot 3^n \\
&= a_n \\
&= \text{LHS}
\end{aligned}$$

■