CSE 16
Spring 2024
Quiz 1 **Solutions**

1. (10 Points) Prove that the *exclusive-or* operator is associative, $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$, by filling in the truth table below.

   Solution:

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$	$(p \oplus q) \oplus r \leftrightarrow p \oplus (q \oplus r)$
0	0	0	**0**	**0**	**0**	**0**	**1**
0	0	1	**0**	**1**	**1**	**1**	**1**
0	1	0	**1**	**1**	**1**	**1**	**1**
0	1	1	**1**	**0**	**0**	**0**	**1**
1	0	0	**1**	**1**	**0**	**1**	**1**
1	0	1	**1**	**0**	**1**	**0**	**1**
1	1	0	**0**	**0**	**1**	**0**	**1**
1	1	1	**0**	**1**	**0**	**1**	**1**

   Since $(p \oplus q) \oplus r \leftrightarrow p \oplus (q \oplus r)$ is a tautology, $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$. ■

2. (10 Points) Let $K(x, y)$ be the statement "x knows y", where the domain for both x and y is the set of all people. Assume it is possible for x to know y while y does not know x, and hence $K(x, y)$ and $K(y, x)$ are different statements. Translate the following expression from first order logic into English.

$$\exists x\ \exists y\ \left[(x \neq y) \land K(x,y) \land K(y,x) \land\ \forall z\left((z \neq x) \land (z \neq y) \rightarrow \left(\neg K(x,z) \land \neg K(y,z)\right)\right)\right]$$

   Solution: **'There are two distinct persons who know each other, and no one else.'**

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3. (20 Points) Show that $[(p \lor q) \land \neg p] \rightarrow q$ is a tautology without using truth tables. Justify each step in your proof by referring to the logical equivalences listed on the attached page.

   Proof:

$[(p \lor q) \land \neg p] \rightarrow q \equiv \neg[(p \lor q) \land \neg p] \lor q$	Table7 #1
$\equiv [\neg(p \lor q) \lor \neg\neg p] \lor q$	DeMorgan
$\equiv [\neg(p \lor q) \lor p] \lor q$	Double Negation
$\equiv \neg(p \lor q) \lor (p \lor q)$	Associative
$\equiv T$	Negation

■

4. (10 Points) Prove that for all $n \in \mathbb{Z}$, if $n^3 + 5$ is even, then n is odd. (Hint: use contraposition.)

Proof:
Let $n \in \mathbb{Z}$. Following the hint, we prove that if n is even, then $n^3 + 5$ is odd. Assume $n = 2k$ for some integer k. It follows that

$$\begin{aligned} n^3 + 5 &= (2k)^3 + 5 \\ &= 8k^3 + 5 \\ &= (8k^3 + 4) + 1 \\ &= 2(4k^3 + 2) + 1, \end{aligned}$$

which is odd, since $4k^3 + 2$ is an integer. Therefore $n^3 + 5$ is odd. By contraposition, we have shown that if $n^3 + 5$ is even, then n is odd. ■