CMPE 16
Homework Assignment 4
Solutions to Selected Problems

1. (4.1) #6
   Let a, b, c and d be integers with $a \neq 0$ and $b \neq 0$. Show that if $a|c$ and $b|d$, then $ab|cd$.

   Proof:
   Assume $a|c$ and $b|d$. Then there exist $k_1, k_2 \in \mathbb{Z}$ such that $c = k_1 a$ and $d = k_2 b$. Therefore

   $$cd = (k_1 a) \cdot (k_2 b) = (k_1 k_2) \cdot ab,$$

   showing that $ab|cd$. ■

2. (4.3) #12
   Prove that for every positive integer n, there are n consecutive composite integers. Hint: Consider the n consecutive integers starting with $(n+1)! + 2$.

   Proof:
   Following the hint, we consider the n consecutive integers

   $$S = \{\, (n+1)! + 2, (n+1)! + 3, \dots \dots, (n+1)! + n, (n+1)! + (n+1) \,\}.$$

   We will show that each integer in S is composite. Since $(n+1)! = 2 \cdot 3 \cdot \cdots \cdots n \cdot (n+1)$ is clearly divisible by each of the integers $2, 3, \dots, n, (n+1)$, we have

   $$\begin{aligned} &2 \mid (n+1)! + 2 \\ &3 \mid (n+1)! + 3 \\ &\ \vdots \\ &\ \vdots \\ &n \mid (n+1)! + n \\ &(n+1) \mid (n+1)! + (n+1). \end{aligned}$$

   Therefore each of the integers in S is composite, as claimed. ■

3. (4.3) #32d
   Use the Euclidean Algorithm to find $\gcd(1529, 14039)$.

   Solution:

   $$\begin{aligned} 14039 &= 1529 \cdot 9 + 278 \\ 1529 &= 278 \cdot 5 + 139 \\ 278 &= 139 \cdot 2 + 0 \end{aligned}$$

   Therefore $\mathbf{gcd(1529, 14039) = 139}$. ■