CMPE 16
Homework Assignment 2
Solutions to Selected Problems

1. (1.7) #18a
   Prove (using contraposition) that for any integer n, if $3n + 2$ is even, then n is even.

   Proof:
   We prove the contrapositive statement: if n is odd, then $3n + 2$ is odd.

   Let $n \in \mathbb{Z}$ and assume n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$. Therefore

$$
\begin{aligned}
3n + 2 &= 3(2k + 1) + 2 \\
&= (6k + 3) + 1 + 1 \\
&= (6k + 4) + 1 \\
&= 2(3k + 2) + 1,
\end{aligned}
$$

   and hence $3n + 2$ is odd. ■

2. (1.8) #16
   Prove that for any $a, b, c \in \mathbb{R}$, with $a \neq 0$, that there exists a unique $x \in \mathbb{R}$ satisfying the equation

$$ax + b = c.$$

   Proof:
   We prove separately (1) existence and (2) uniqueness of x satisfying the above equation.

   (1) Let $x = (c - b)/a$. Then

$$ax + b = a\left(\frac{c - b}{a}\right) + b = (c - b) + b = c$$

   showing that at least one solution exists.

   (2) Suppose that both x_1 and x_2 satisfy the above equation. Then both

$$ax_1 + b = c \text{ and } ax_2 + b = c.$$

   Thus $ax_1 + b = ax_2 + b$, whence $ax_1 = ax_2$, and therefore $x_1 = x_2$, showing that the solution is unique.

■

3. <u>(2.1) #22</u>
Prove or disprove: for any sets A and B, if $\mathbb{P}(A) = \mathbb{P}(B)$, then $A = B$.

Solution:
The statement is **true**, as we now prove.

Proof:
Let A and B be sets, and assume $\mathbb{P}(A) = \mathbb{P}(B)$. Let $x \in A$. Then $\{x\} \subseteq A$, and hence $\{x\} \in \mathbb{P}(A)$. Since $\mathbb{P}(A) = \mathbb{P}(B)$, we have $\{x\} \in \mathbb{P}(B)$, hence $\{x\} \subseteq B$, and $x \in B$. We've shown $x \in A \rightarrow x \in B$. Repeating the same argument with A and B interchanged gives $x \in B \rightarrow x \in A$. Therefore $x \in A \leftrightarrow x \in B$, which proves that $A = B$. ■