CMPE 16
Homework Assignment 1
Solutions to Selected Problems

1. (1.3) #62a
   Determine whether the following compound proposition is satisfiable:

$$(p \lor q \lor \neg r) \land (p \lor \neg q \lor \neg s) \land (p \lor \neg r \lor \neg s) \land (\neg p \lor \neg q \lor \neg s) \land (p \lor q \lor \neg s)$$

Solution:
The above expression is **satisfiable**. In fact, of the 16 possible assignments of truth values to the propositional variables (p, q, r, s), there are exactly 9 that will make the expression true. These assignments are listed below.

$(0, 0, 0, 0)$
$(0, 1, 0, 0)$
$(0, 1, 1, 0)$
$(1, 0, 0, 0)$
$(1, 0, 0, 1)$
$(1, 0, 1, 0)$
$(1, 0, 1, 1)$
$(1, 1, 0, 0)$
$(1, 1, 1, 0)$

Note to grader: At least one of the above 4-tuples of truth values must be mentioned to get full credit.

■

2. (1.5) #10j
   Let $F(x, y)$ be the statement 'x can fool y', where the domain consists of all people. Use first order logic to express the following statement.

'There is a person who can fool exactly one person other than himself'

Solution:

$$\exists x\, \exists y\, \big[(x \neq y) \land F(x, y) \land \forall z\big((z \neq x) \land (z \neq y) \;\rightarrow\; \neg F(x, z)\big)\big]$$

■

3. (1.5) #34
   Find a common domain for the variables x, y and z for which the statement

$$\forall x\, \forall y\, \big[(x \neq y) \rightarrow \forall z\, \big((z = x) \lor (z = y)\big)\big]$$

is true, and another domain for which it is false.

Solution:
The statement is **true** over the domain $U = \{1,2\}$, and it is **false** over the domain $U = \{1, 2, 3\}$.

In general, the above statement will be **true** if the universe of discourse contains **at most 2 elements**, and **false** if it contains **at least 3 elements**. ■