CSE 16 4-18-24

- Quiz 1 today

  3:20 - 4:20 lecture

  4:25 - 4:55 Quiz 1

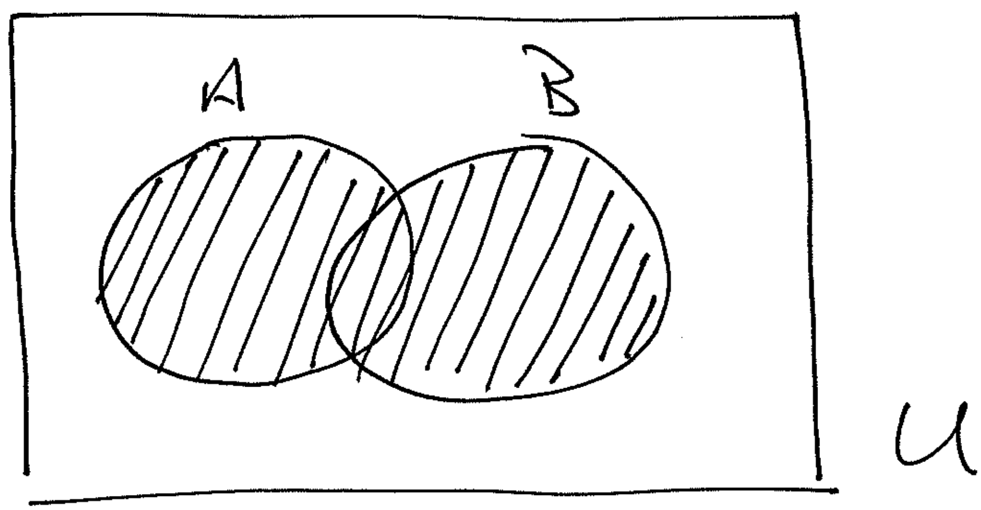
- hw1 : review Period
- hw2 : Posted
- Lab1 : due ~~Sat~~ Fri. 10:00 PM

2.2 Set operations

Defn

The union of sets A, B is

$$A \cup B = \{x \in U \mid x \in A \lor x \in B\}$$

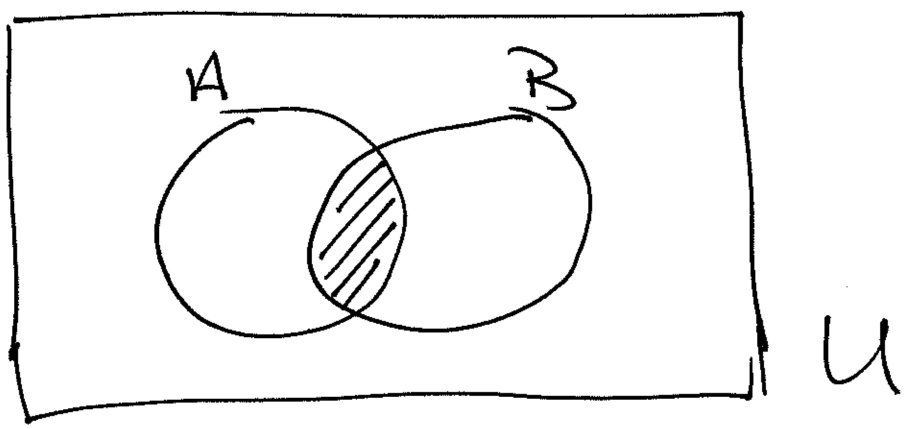


$A \cup B$

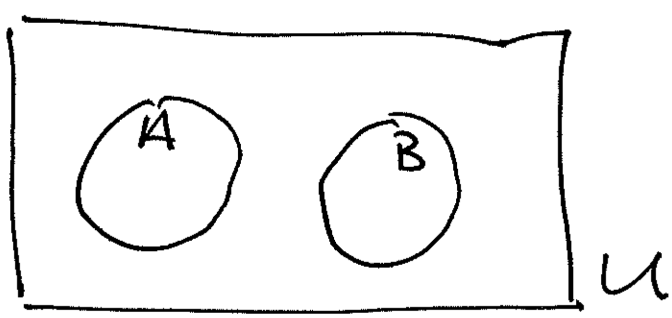
<u>Defn</u>

The intersection of $A, B \subseteq U$

$$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$$



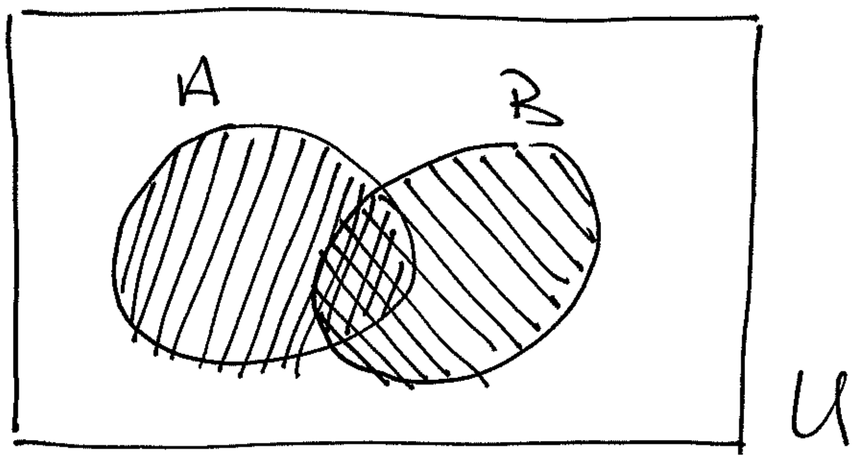
<u>Defn</u> We say A, B are <u>disjoint</u> iff $A \cap B = \emptyset$



Theorem

If A and B are finite, then so are $A \cup B$ and $A \cap B$

$$|A \cup B| = |A| + |B| - |A \cap B|$$



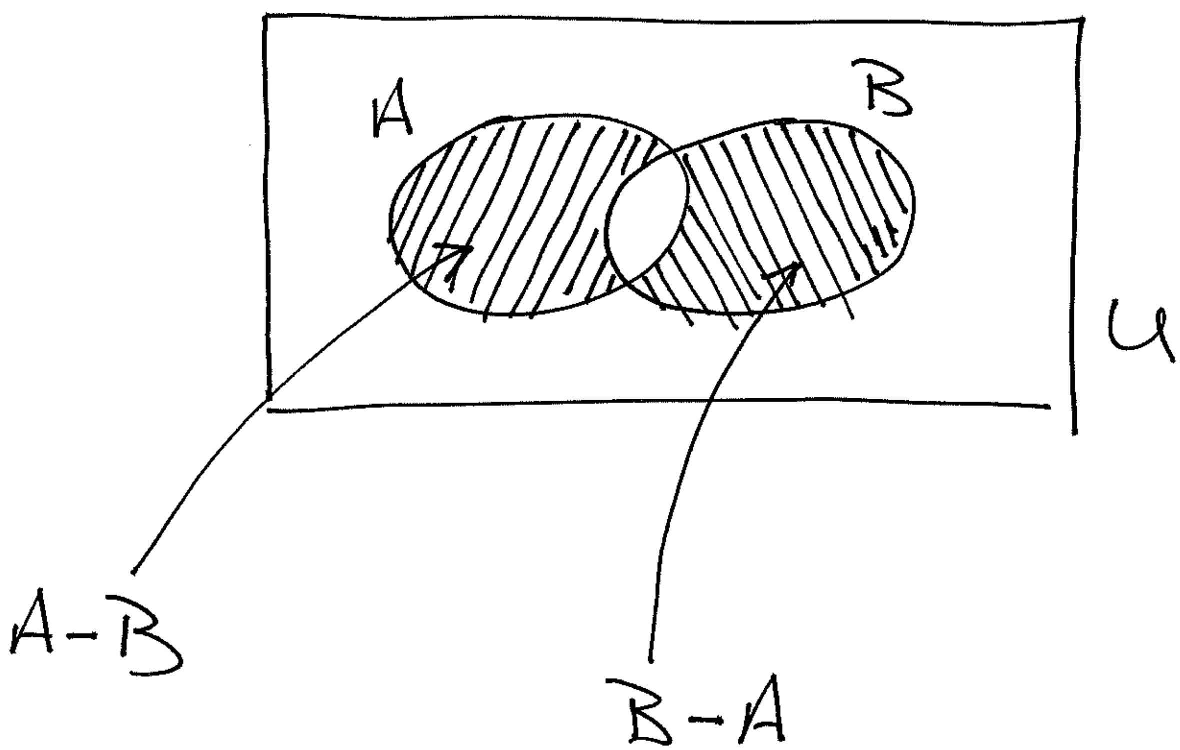
Generalizes to: PIE

Principle of Inclusion-Exclusion.

<u>Defn</u>

The <u>set difference</u> is

$$A - B = \{x \in U \mid x \in A \wedge x \notin B\}$$

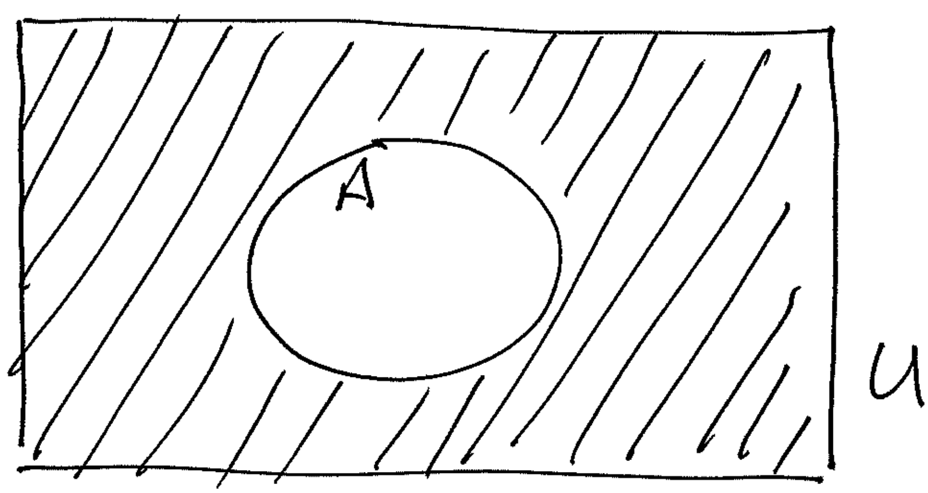


<u>note</u>: $(A-B) \cap (B-A) = \phi$

Defn

The complement of A is

$$\bar{A} = U - A = \{x \in U \mid x \notin A\}$$



Set Identities (Table 1. P.130)

- Identity: $A \cap U = A$

  $A \cup \emptyset = A$

- Domination: $A \cup U = U$

  $A \cap \emptyset = \emptyset$

- Idempotent: $A \cup A = A$

  $A \cap A = A$

- Double Complement: $\overline{\overline{A}} = A$

- Commutative: $A \cup B = B \cup A$

  $A \cap B = B \cap A$

- Associative: $(A \cup B) \cup C = A \cup (B \cup C)$

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- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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- Demorgan: $\overline{A \cup B} = \bar{A} \cap \bar{B}$

  $\overline{A \cap B} = \bar{A} \cup \bar{B}$

- Absorbtion: $A \cup (A \cap B) = A$

  $A \cap (A \cup B) = A$

- Complement: $A \cup \bar{A} = U$

  $A \cap \bar{A} = \phi$

<u>Ex.</u> Prove 1st Demorgan. (logic)

$$\overline{A \cup B} = \{x \in U \mid x \notin A \cup B\}$$

$$= \{x \in U \mid \neg (x \in A \cup B)\}$$

$$= \{x \in U \mid \neg (x \in A \lor x \in B)\}$$

$$= \{x \in U \mid \neg (x \in A) \land \neg (x \in B)\}$$

$$= \{x \in U \mid x \notin A \land x \notin B\}$$

$= \{x \in U \mid x \in \bar{A} \wedge x \in \bar{B}\}$

$= \bar{A} \cap \bar{B}$. $\blacksquare$

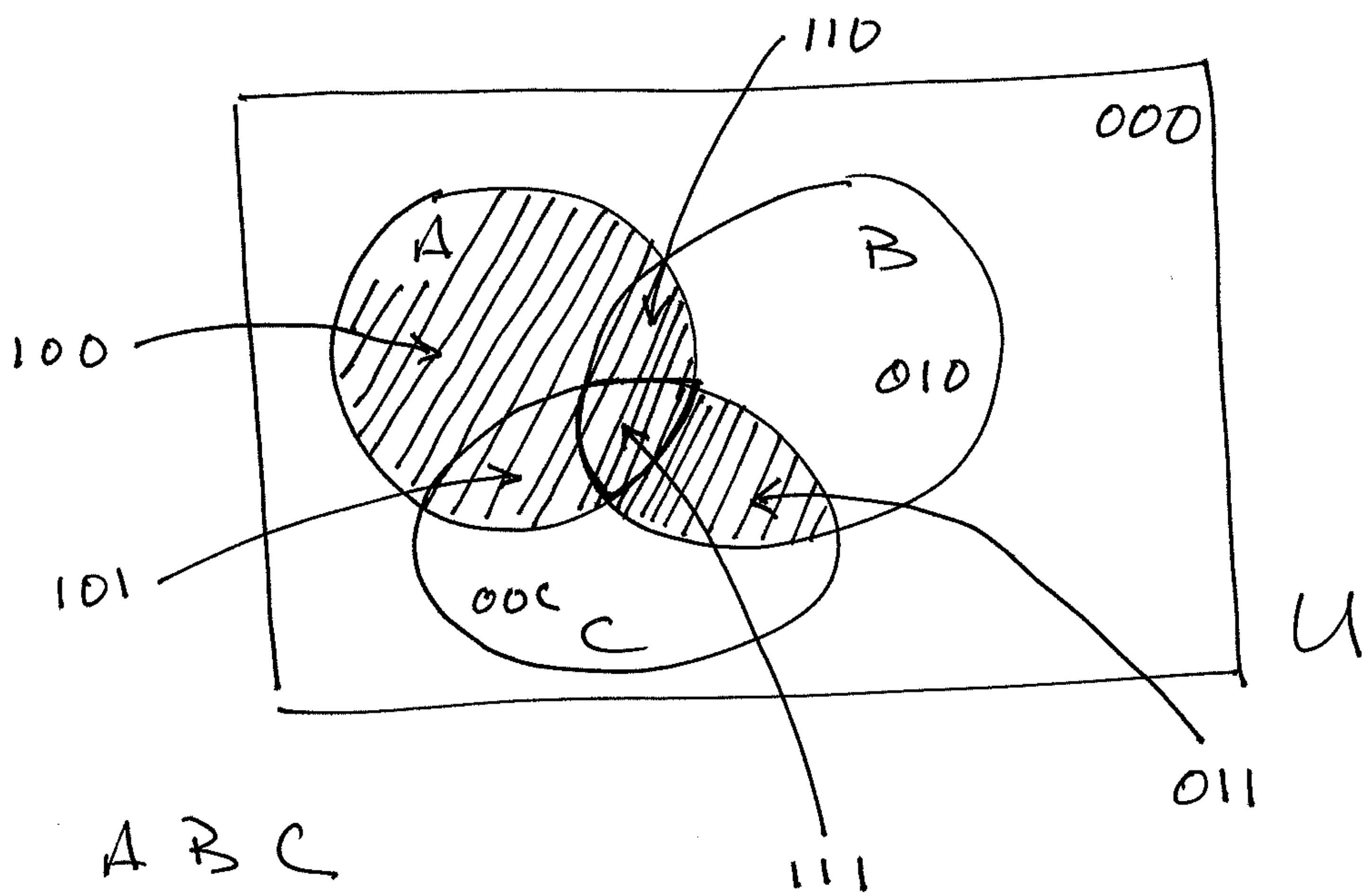
Ex. 2nd Demorgan (membership table)

A	B	$A \cap B$	$\overline{A \cap B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$.

Ex. Distributive 1^{st}

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

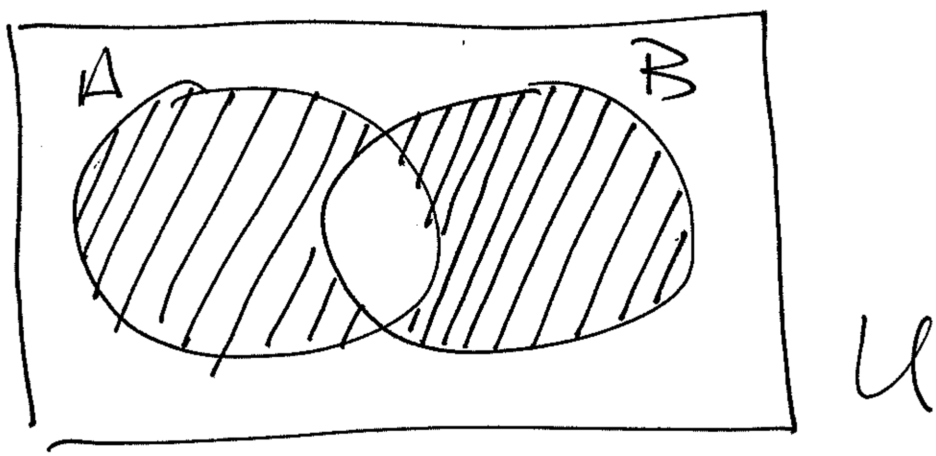


A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Defn

The symmetric difference of A and B

$$A \oplus B = \{ x \in U \mid x \in A \oplus x \in B \}$$



Exercise show

- $A \oplus B = (A - B) \cup (B - A)$
- $A \oplus B = (A \cup B) - (A \cap B)$

Representation of sets

one way: start with a fixed universe of objects

$$U = \{x_1, x_2, x_3, \ldots, x_n\}$$

Any subset of U is represented as a bit string of length n

$$b_1 b_2 b_3 \ldots b_n$$

where $b_i = \begin{cases} 0 & \text{if } x_i \text{ not in subset} \\ 1 & \text{if } x_i \text{ is in subset} \end{cases}$

Ex. $\mathcal{U} = \{1, 2, 3, 4\}$

$\{1, 3, 4\}$ — 1011

$\{2, 3\}$ — 0110

ϕ — 0000

$\mathcal{U}$ — 1111

Then

Union — bitwise $\vee$

intersection — bitwise $\wedge$

sym. diff → " exor