CSE 107
Homework Assignment 1

Section 1.2:

1. We roll two fair 4-sided dice. Let the outcome of the experiment be (r_1, r_2) where r_1 and r_2 are the numbers showing on top of the two dice, respectively. Assume that all possible outcomes have equal probability. Find the probability that:

   (a) r_1 is odd.
   (b) Both r_1 and r_2 are odd.
   (c) $r_1 + r_2 > 6$.
   (d) $r_1 < r_2$

2. You enter a special kind of chess tournament, in which you play one game against each of three opponents, but you get to choose the order in which you play your opponents. You win the tournament if you win two games in a row. You know your probability of a win against each of the three opponents, and that the opponents are of differing strengths. Label the players 1, 2 and 3 by increasing strength, so that player 1 is the weakest and player 3 the strongest. Let p_i be your probability of winning against player i, so that $p_1 > p_2 > p_3$. Assuming that individual matches are independent events, determine the optimal order(s) of play: 123, 132, 213, 231, 312, or 321. Determine your probability of winning the tournament, given the optimal order of play.

3. Alice and Bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

   A: The magnitude of the difference of the two numbers is greater than 1/3.
   B: At least one of the numbers is greater than 1/3.
   C: The two numbers are equal.
   D: Alice's number is greater than 1/3.

   Find the probabilities

   (a) $P(A)$
   (b) $P(B)$
   (c) $P(A \cap B)$
   (d) $P(C)$
   (e) $P(D)$
   (f) $P(A \cap D)$

Section 1.3:

4. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

   (a) Find the probability that doubles were rolled.
   (b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

(c) Find the probability that at least one die is a 6.
(d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.

5. A coin is tossed twice. It is claimed that the probability of getting two heads, given that the first toss is a head, is greater than or equal to the probability of getting two heads, given that at least one of the tosses is a head. Determine whether or not this claim is true. [Hint: Let A be the event that the first toss is a head, and B the event that the second toss is a head. Compare the conditional probabilities $P(A \cap B \mid A)$ and $P(A \cap B \mid A \cup B)$.]

Section 1.4:

6. A magnetic tape storing information in binary form has been corrupted, so it can only be read with bit errors. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?

7. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 85% accurate if the student is in fact overstressed. It is known that 99.5% of all students are overstressed. Given that a particular student tests negative for stress, what is the probability that the test results are correct, and that this student is not overstressed?

8. A hiker starts by taking one of n available trails, denoted $1, 2, \ldots, n$. An hour into the hike, trail i subdivides into $1 + i$ subtrails, only one of which leads to the hiker's destination. The hiker has no map and makes random choices of trail and subtrail. What is the probability of reaching the destination?