CSE 107

2-26-26

Recall:

$X_1, \dots, X_n$ are i.i.d r.v.

common distribution is $f_X(x)$

N discrete r.v. range $\subseteq \{0, 1, \dots\}$

$\{N, X_1, \dots\}$ are indep.

$$Y = X_1 + \dots + X_N$$

Goal: find $E[Y]$, $Var(Y)$

We found

- $E[Y] = E[N] \cdot E[X]$
- $Var(Y) = E[N] \cdot Var(X) + E[X]^2 \cdot Var(N)$

Ex

suppose have 2 coins

coin 1 : $P(\text{head}) = p$

coin 2 : $P(\text{head}) = q$

- Flip coin 1 indep. until 1^{st} head

  let $N = \#$flips.

  $\therefore N$ is Geometric(p)

- Flip coin 2 N times.

  let $Y = \#$heads

  $\therefore Y$ is Binomial(q, N)

Thus

$$Y = X_1 + X_2 + \dots + X_n$$

where each X_i is Bernoulli(q)

let $E[X]$, $Var(X)$ be common mean & Var. of X_i

We have

$$E[N] = \frac{1}{p}, \quad Var(N) = \frac{1-p}{p^2}$$

and

$$E[X] = q, \quad Var(X) = q(1-q)$$

Thus

$$E[Y] = E[N] \cdot E[X] = \boxed{\frac{q}{p}}$$

Also

$$Var(Y) = E[N] \cdot Var(X) + E[X]^2 \cdot Var(N)$$

$$= \frac{q(1-q)}{p} + \frac{q^2(1-p)}{p^2}$$

$$= \boxed{\frac{q}{p}\left((1-q) + \frac{q}{p}(1-p)\right)}$$

Notation

write $X \sim Y$ to mean $f_X = f_Y$. Equivalently $F_X = F_Y$.

Theorem

let $X, Y \sim \text{Uniform}(0,1)$, be independent. Then

$$\max(X,Y) \sim \sqrt{X}$$