CSE 107 1-6-26

1.1 Sets

Read this.

1.2 Probabilistic Models

A Probability Model consists of

(1) A set Ω, called sample space. The elements $\omega \in \Omega$ are called outcomes.

A subset $A \subseteq \Omega$ is called an <u>event</u>.

(2) A <u>probability law</u> $P(A)$ that assigns a real number to every event, called the probability of A.

<u>Axioms for $P()$</u>

(i) $P(A) \geq 0$ for all $A \subseteq \Omega$

(ii) $P(\Omega) = 1$

(iii) Additivity

- if $A, B \subseteq \Omega$ and $A \cap B = \phi$ (A, B are disjoint), then

$$P(A \cup B) = P(A) + P(B)$$

- if $A_1, A_2, \dots, A_n \subseteq \Omega$ is a finite sequence of pairwise disjoint events ($A_i \cap A_j = \phi$), then

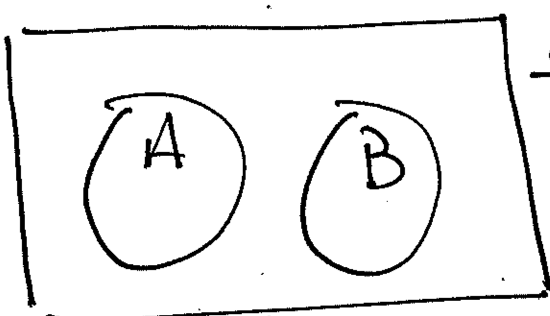
$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

- if $A_1, A_2, \ldots\ldots$ is an infinite sequence of pairwise disjoint events $(A_i \cap A_j = \phi)$, then

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots\ldots$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$



Ω

$A \cap B = \phi$

$$P(A \cup B) = P(A) + P(B)$$

Ex. flip a fair coin

$$\Omega = \{H, T\}$$

note: $\{H\} = H$, $\{T\} = T$

since $\Omega = \{H\} \cup \{T\}$ and

$\{H\} \cap \{T\} = \phi$, axioms (iii), (ii)

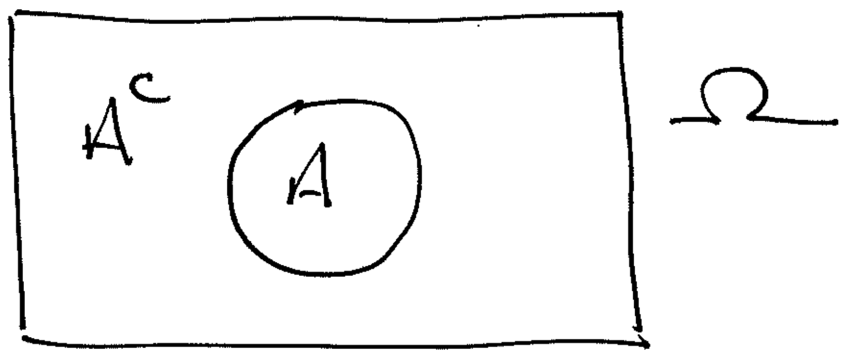
give

$$1 = P(\Omega) = P(H) + P(T)$$

also, fair means $P(H) = P(T)$

$$\therefore P(H) = \frac{1}{2}, \; P(T) = \frac{1}{2}$$

In General, if $A \subseteq \Omega$, then $A \cup A^c = \Omega$ and $A \cap A^c = \phi$



$\therefore \quad 1 = P(\Omega) = P(A) + P(A^c)$

Hence $P(A) \leq 1$ for any $A \subseteq \Omega$

Thus

$$0 \leq P(A) \leq 1$$

$\square$

Also $P(A)+P(A^c)=1$ gives

$$\boxed{P(A^c) = 1 - P(A)}$$

and

$$\boxed{P(A) = 1 - P(A^c)}$$

Evidently $P(\;)$ is a function

$$P : \underbrace{\mathbb{P}(\Omega)}_{\substack{\text{set of all}\\ \text{subsets}\\ \text{of } \Omega}} \longrightarrow [0,1]$$

Also

$$
\begin{aligned}
1 &= P(\Omega) \\
&= P(\Omega \cup \Omega^c) \\
&= P(\Omega \cup \emptyset) \\
&= P(\Omega) + P(\emptyset) \\
&= 1 + P(\emptyset)
\end{aligned}
$$

$\therefore \; P(\emptyset) = 0$

Previous example: $\Omega = \{H, T\}$

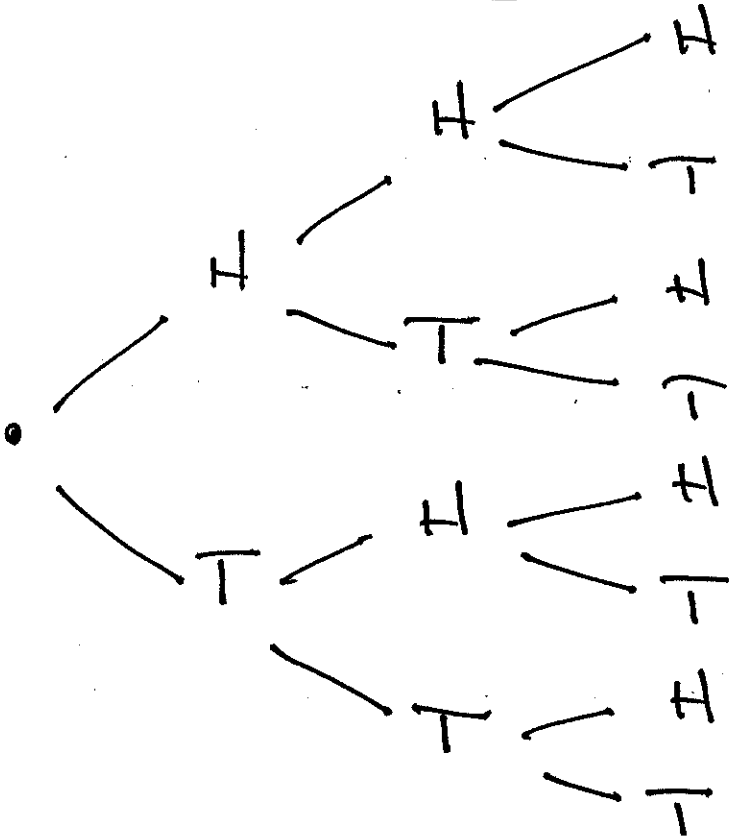
Events: $\Omega, \{H\}, \{T\}, \phi$

Probabilities: $1, \frac{1}{2}, \frac{1}{2}, 0$

<u>Ex.</u> flip 3 times

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Tree diagram



Outcome	Prob.
HHH	$\frac{1}{8}$
⋮	⋮
THH	$\frac{1}{8}$
⋮	⋮
TTT	$\frac{1}{8}$