

CSE 107
Homework Assignment 5

Section 3.2: Cumulative Distribution Functions

1. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Your score in test i , where $i = 1, 2, 3$, takes one of the values from i to 10 with equal probability $1/(11 - i)$, independently of the scores in other tests. What is the PMF of the final score?

Section 3.3: Normal Random Variables

2. A radar tends to overestimate the distance of an aircraft, and the error is a normal random variable with a mean of 50 meters and a standard deviation 100 meters. What is the probability that the measured distance will be smaller than the true distance?
3. Let X be normal with mean 1 and variance 4, and let $Y = 2X + 3$.
 - a. Calculate the PDF of Y .
 - b. Find $P(Y \geq 0)$.
4. A signal of amplitude $s = 2$ is transmitted from a satellite and is corrupted by noise. The received signal is $X = s + W$, where the noise W is normal. When the weather is good, W has mean 0 and variance 1. When the weather is bad, W has mean 0 and variance 4. Good and bad weather are equally likely. In the absence of any weather information, determine the following.
 - a. The PDF of X .
 - b. The probability that X is between 1 and 3.
5. A transmitter sends the signals -1 and $+1$, with probabilities p and $1 - p$, respectively. The communication channel introduces additive noise that is normal with mean 0 and variance σ^2 . This noise is independent of the particular signal sent. The receiver at the other end of the channel receives a signal which is the sum of the transmitted signal and the channel noise.
 - a. Let a be a constant between -1 and $+1$. The receiver interpretes its signal as -1 if it is less than a , and as $+1$ if it is greater than a . Find a formula (in terms of a , p and σ) for the probability that the receiver makes an error.
 - b. Determine the probability that the receiver makes an error if $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.

Section 3.4: Joint PDFs of Multiple Continuous Random Variables

6. A point is chosen at random (according to a uniform PDF) within a semicircle S of the form

$$S = \{(x, y) | x^2 + y^2 \leq r^2, y \geq 0\}$$

for some $r > 0$.

- a. Find the joint PDF $f_{X,Y}(x, y)$ of the coordinates X and Y of the chosen point.
- b. Find the marginal PDF $f_Y(y)$ and use it to find $E[Y]$.
- c. Find $E[Y]$ directly (without using $f_Y(y)$) by using the formula $E[Y] = \iint_S y f_{X,Y}(x, y) dx dy$.
Hint: use polar coordinates to compute this integral.