CSE 107
Homework Assignment 2

<u>Section 1.5</u>: Independence

1. A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are 1×1.5 inches, the faces showing 2 or 5 are 1×0.4 inches, and the faces showing 3 or 4 are 0.4×1.5 inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?

2. A parking lot consists of a single row containing n parking spaces ($n \geq 2$). Mary arrives when all spaces are free. Tom is the next person to arrive. Each person makes an equally likely choice among all available spaces at the time of arrival. Describe the sample space. Obtain P(A), the probability the parking spaces selected by Mary and Tom are at most 2 spaces apart.

3. A particular jury consists of 7 jurors. Each juror has a 0.2 chance of making the wrong decision, independently of the others. If the jury reaches a decision by majority rule, what is the probability that it will reach a wrong decision?

4. Suppose that events A, B, and C are independent. Use the definition of independence to show that A and $B \cup C$ are independent.

5. Three persons each independently roll a fair n-sided die. Let A_{ij} be the event that person i and person j roll the same number. Show that the events A_{12}, A_{13} and A_{23} are pairwise independent but are not independent.

<u>Section 1.6 Counting</u>:

6. A candy factory has an endless supply of red, orange, yellow, green, blue, and violet jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each. One possible color distribution, for example, is a jar of 58 red, 22 yellow, and 20 green jelly beans. As a marketing gimmick, the factory guarantees that no two jars have the same color distribution.

   (a) What is the maximum number of jars the factory can produce?
   (b) Suppose the factory sells all of the jars you counted in (a). What is the probability that a randomly purchased jar contains no violet jelly beans?

<u>Section 2.2</u>: Probability mass functions

7. The annual premium of a special kind of insurance starts at \$1000 and is reduced by 10% after each year where no claim has been filed. The probability that a claim is filed in a given year is 0.05, independently of preceding years. What is the PMF of the total premium paid up to and including the year when the first claim is filed?

Section 2.3: Functions of a random variable

8. Let X be a discrete random variable that is uniformly distributed over the set of integers in the range $[a, b]$, where a and b are integers with $a < 0 < b$. Find the PMF of the random variables $\max(0, X)$ and $\min(0, X)$.

9. Let X be a discrete random variable, and let $Y = |X|$.

   (a) Assume that the PMF of X is

   $$p_X(x) = \begin{cases} cx^2 & \text{if } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

   where c is a suitable constant. Determine the value of c.

   (b) For the PMF of X given in part (a) calculate the PMF of Y.

   (c) Give a general formula for the PMF of $Y = |X|$ in terms of the PMF of X.