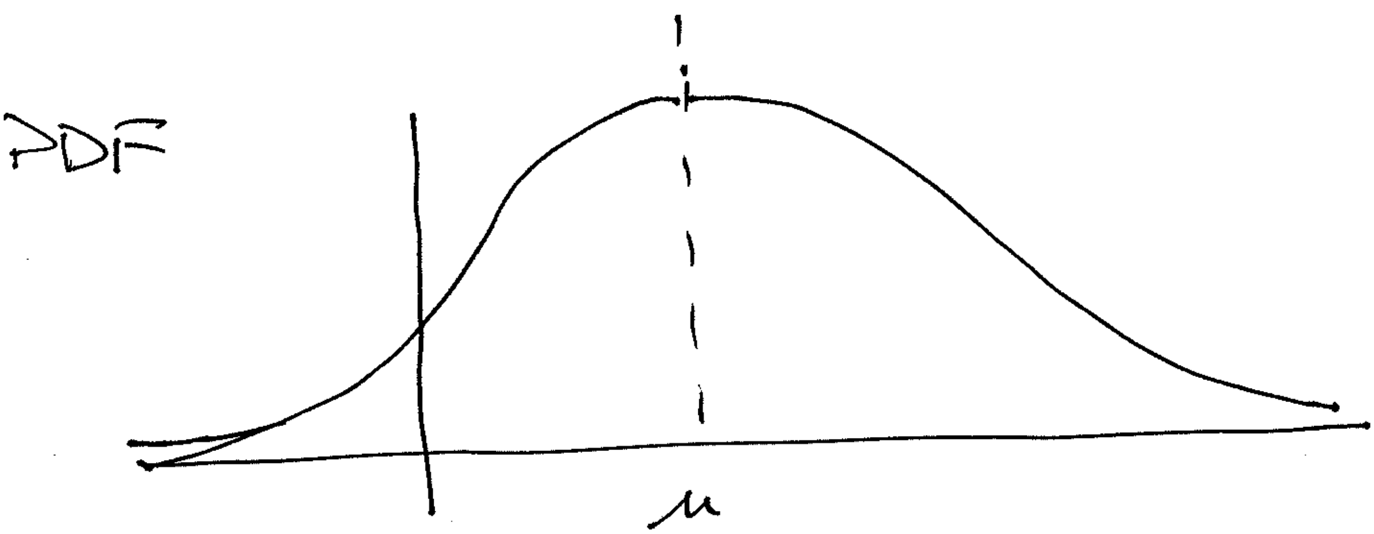
ese 107 2-10-25

Note: $f_X(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

is invariant under

$$(x-\mu) \longleftrightarrow -(x-\mu)$$



so $f_X(x)$ is symmetric about $x=\mu$,

hence $E[X]=\mu$.

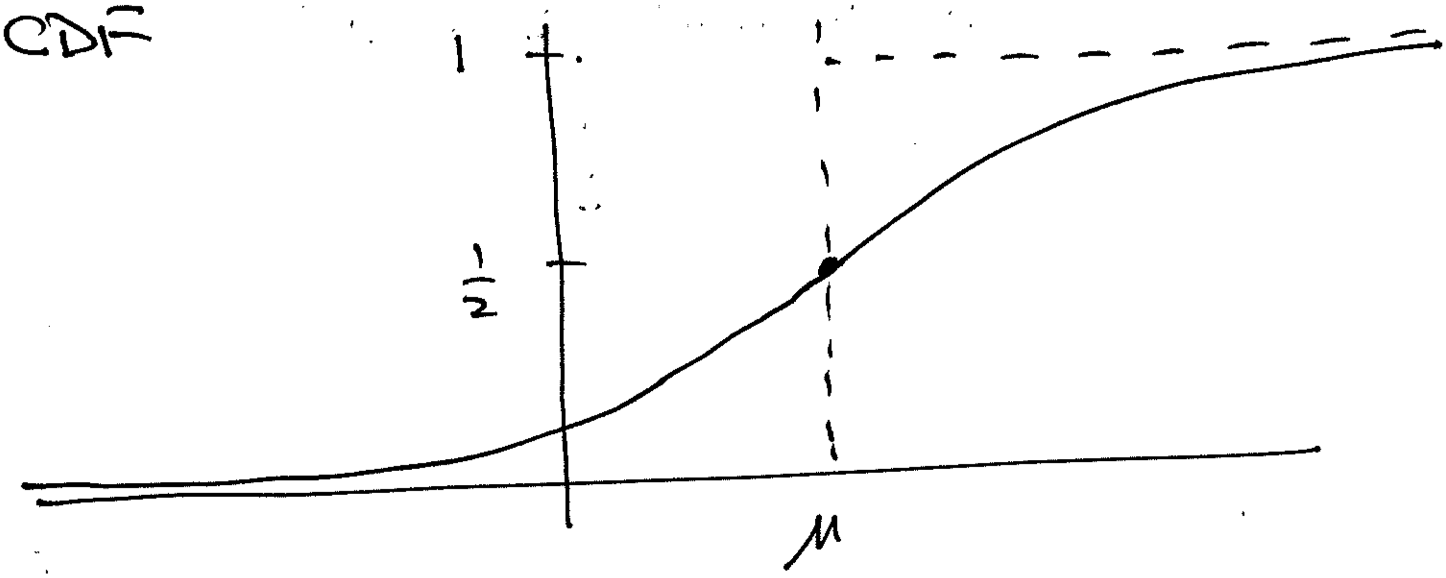
Lemma

if $f_X(x) = f_X(-x)$, for all $x \in \mathbb{R}$,

then $E[X] = 0$.

Proof: exercise.

CDF



<u>Theorem</u>

If X is a normal r.v. with mean & variance μ, σ^2, resp. and if $a \neq 0$, then

$$Y = aX + b$$

is also normal with

$$E[Y] = a\mu + b$$

and

$$Var(Y) = a^2 \sigma^2.$$

<u>Proof</u>: later

Defn

A normal r.v. Y is called the <u>standard normal</u> iff it has

$\mu = 0$, $\sigma = 1$.

PDF: $$f_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}}$$

Notation for CDF of std. normal

$$\Phi(y) = F_Y(y)$$
$$= P(Y \le y) = P(Y < y)$$
$$= \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}}\, dt$$

Handout: <u>standard normal table</u>

gives values of $\Phi(y)$ for

$$0 \le y \le 3.49$$

If $y > 3.49$, then $\Phi(y) \approx 1$. The identity

$$\Phi(-y) = 1 - \Phi(y) \qquad (y \in \mathbb{R})$$

gives us $\Phi(y)$ for $y < 0$.

Ex.

- $\Phi(1.84) = .9671$

- $\Phi(-1.84) = 1 - \Phi(1.84)$

$$= 1 - (.9671)$$

$$= .0329$$

How to compute values for a non-standard normal: X

Let

$$Y = \frac{X - \mu}{\sigma} = \frac{1}{\sigma} X - \frac{\mu}{\sigma}$$

where $E[X] = \mu$, $Var(X) = \sigma^2$.

Then Y is normal with

$$E[Y] = \frac{1}{\sigma}\mu - \frac{\mu}{\sigma} = 0$$

and

$$Var(Y) = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$$

$\therefore$ Y is standard normal !!

Thus

$$F_X(x) = P(X \le x)$$
$$= P\left(\frac{X-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right)$$
$$= P\left(Y \le \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Ex.

The mean high temp. in santa cruz on Nov. 1 is 66° F with std. dev. 4.4°. What is the prob. that the high will be ≤ 57°.

Solu

let X be the high temp.

$$P(X \le 57) = P\left(\frac{X-66}{4.4} \le \frac{57-66}{4.4}\right)$$

$$= P(Y \le -2.04)$$

$$= \Phi(-2.04)$$

$$= 1 - \Phi(2.04)$$

$$= 1 - .9793 = .0207$$