cse 107 1-8-25 $\boxed{1}$

⋮

let $A = \Omega$ (in $A \cup A^c = \Omega$

so $P(A) + P(A^c) = 1$)

$$1 = P(\Omega) \qquad \emptyset$$

$$= P(\Omega \cup \Omega^c)$$

$$= P(\Omega) + P(\emptyset)$$

$$= 1 + P(\emptyset)$$

$\therefore$ $P(\emptyset) = 0$

Back to $\Omega = \{H, T\}$

events: $\Omega, \{H\}, \{T\}, \phi$

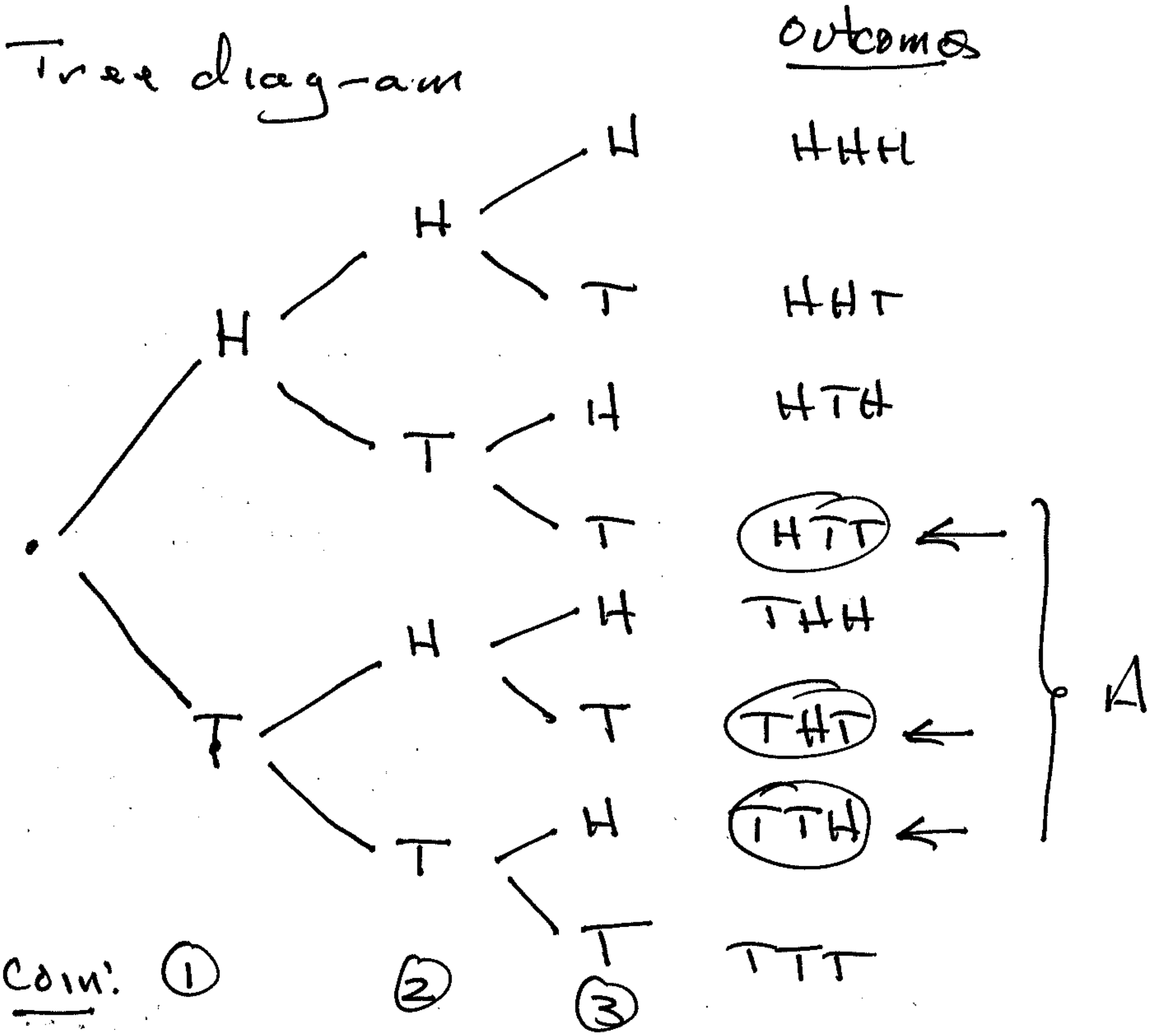
Probabilities: $1, \frac{1}{2}, \frac{1}{2}, 0$

Ex. flip 3 fair coins:

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

'fair': $\frac{1}{8}, \frac{1}{8}, \frac{1}{8} \ldots \ldots \frac{1}{8}$

Tree diagram



Let $A = \{\text{exactly 1 head}\}$

$= \{HTT, THT, TTH\}$

$= \{HTT\} \cup \{THT\} \cup \{TTH\}$

$\therefore P(A) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

Generalize:

Discrete Probability law

If Ω is is finite, the $P(\cdot)$ is specified by the probabilities of each $\omega \in \Omega$! i.e. if you know $P(\omega)$ for all $\omega \in \Omega$, Then you know Prob. of <u>any</u> event $A \subseteq \Omega$. Thus if

$$A = \{\omega_1, \omega_2, \ldots, \omega_k\}$$

then

$$P(A) = P(\omega_1) + P(\omega_2) + \cdots + P(\omega_k).$$

Special case:

Discrete Uniform Prob. Law

If Ω is finite and all $\omega \in \Omega$ are equally likely, then

$$P(A) = \frac{|A|}{|\Omega|}$$

Ex. 2 fair dice are thrown.

sum = 2 3 4 5 6 7

	11	12	13	14	15	16	
	21	22	23	24	25	26	8
	31	32	33	34	35	36	9
{max = 4} →	41	42	43	44	45	46	10
	51	52	53	54	55	56	11
	61	62	63	64	65	66	12

$$P(\text{sum} = 2) = \frac{1}{36}$$

$$P(\text{sum} = 3) = \frac{2}{36}$$

$$4) = \frac{3}{36}$$

$\vdots$

check

$P(\text{sum is even}) = \frac{18}{36} = \frac{1}{2}$

$P(\text{" " odd}) = \frac{1}{2}$

$P(\text{max is } 4) = \frac{7}{36}$

$P(\text{min is } 4) =$

$P(1^{st} \text{ roll} > 2^{nd} \text{ roll}) =$

$P(1^{st} = 2^{nd}) =$

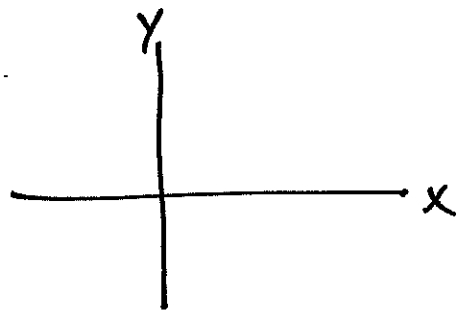
$P(\text{at least die is } 3) =$

continuous models

Ω is a continuum, like

$\mathbb{R}$ $\longleftrightarrow$

$\mathbb{R} \times \mathbb{R}$



Ex.

R and J have a date, each will be delayed by an amount in range 0 to 1 hours. The 1st to arrive will wait 15 minutes, then leave.

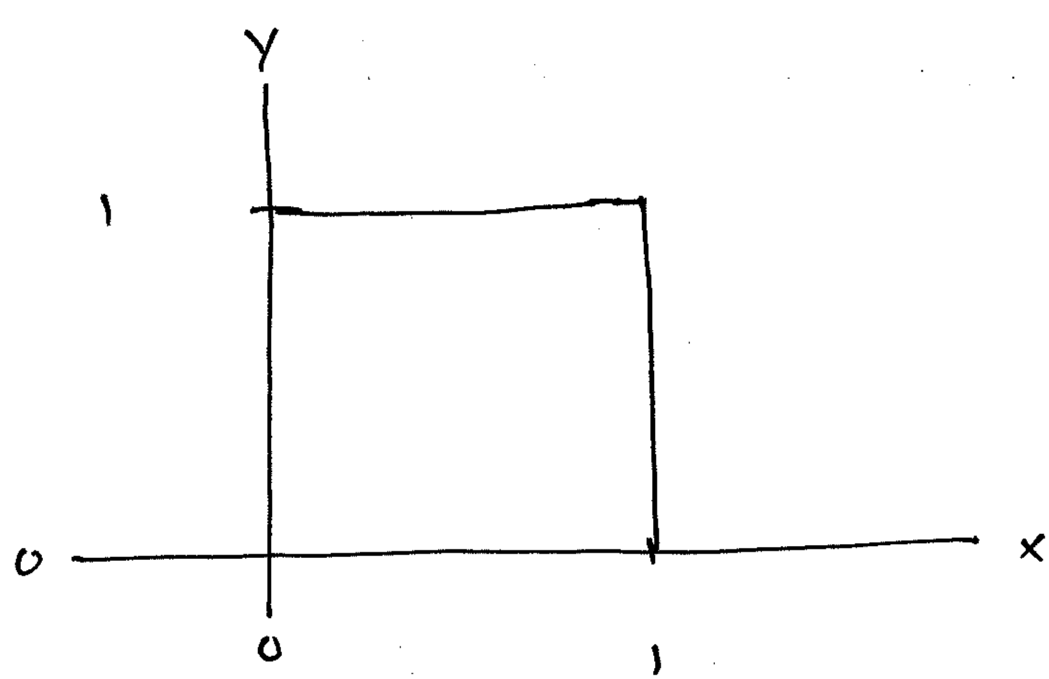
Find $P(R \& J \text{ meet})$, Let

X = R's delay time

Y = J's " "

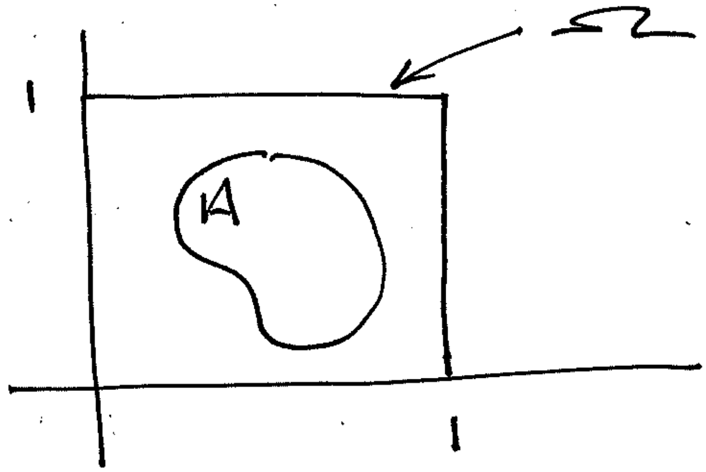
so $0 \leq x \leq 1$ and $0 \leq y \leq 1$

We assume all delay pairs are <u>equally likely</u>.



'fair', 'equally likely', 'random'

'uniform' means



$$P(A) = \underbrace{\text{const}}_{\substack{\text{in this} \\ \text{case} = 1}} \cdot \text{area}(A)$$

note

$$P(\Omega) = \text{const} \cdot \text{area}(\Omega)$$

$$1 = \text{const} \cdot 1$$

$$\therefore \boxed{\text{const} = 1}.$$

Note: for any $(x,y) \in [0,1] \times [0,1]$,

$$P((x,y)) = 0$$

since area of a point is 0.

We seek $P(M)$ where

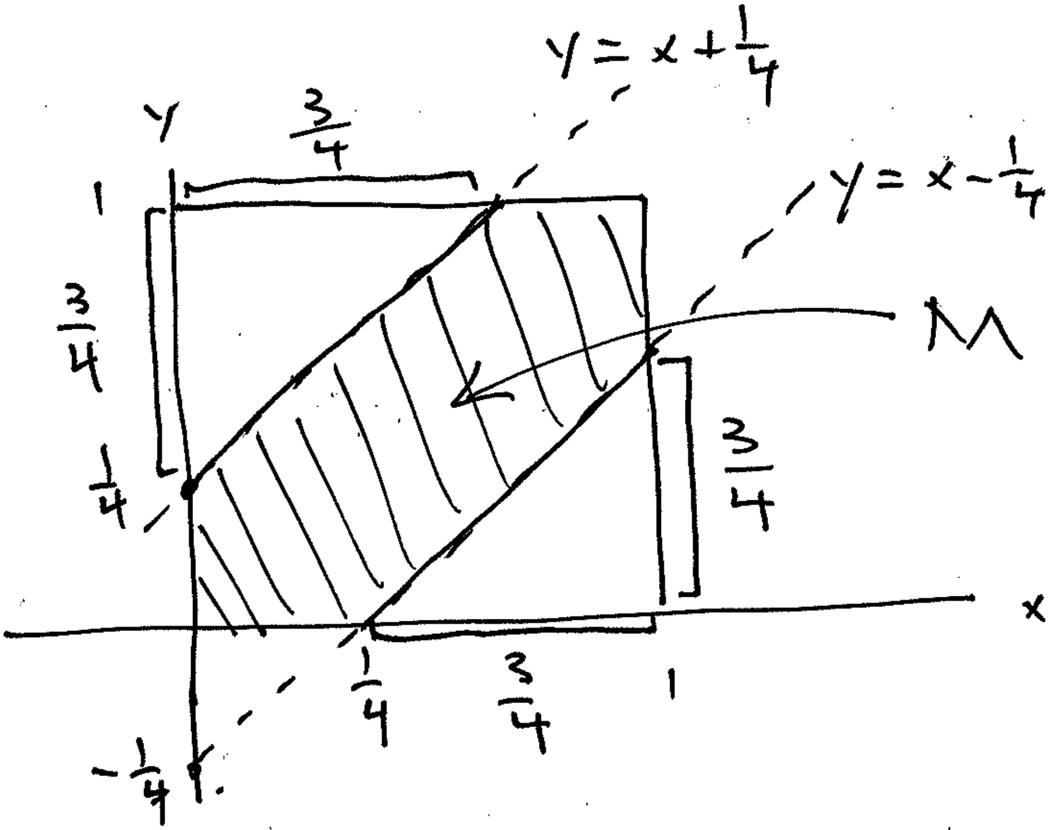
$$M = \{(x,y) \in \Omega \mid |x-y| \le \tfrac{1}{4}\}$$

note: $|x-y| \le \frac{1}{4}$

iff $-\frac{1}{4} \le x-y \le \frac{1}{4}$

iff $y \le x + \frac{1}{4}$ <u>and</u> $y \ge x - \frac{1}{4}$

>0



$$P(M) = \text{area}(M)$$

$$= 1 - \left(\frac{3}{4}\right)^2$$

$$= 1 - \frac{9}{16} = \boxed{\frac{7}{16}}$$

In general

$$1 = P(\Omega) = c \cdot \text{area}(\Omega)$$

$$\therefore \quad c = \frac{1}{\text{area}(\Omega)}$$

Properties of $P(\cdot)$

(a) If $A \subseteq B$, then $P(A) \leq P(B)$

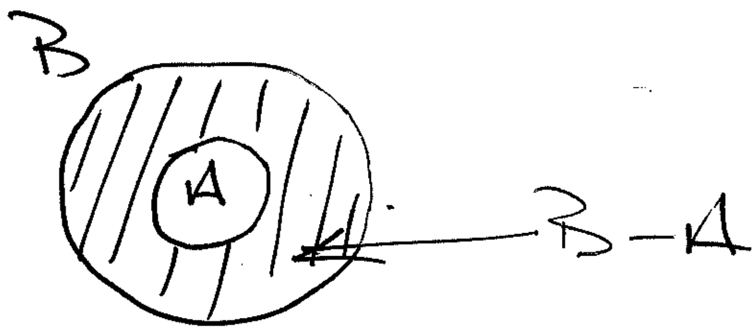
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

✓ (c) $P(A \cup B) \leq P(A) + P(B)$

(d) $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$.

Proof of (a)

$$A \subseteq B \Rightarrow \begin{cases} B = A \cup (B-A) \\ A \cap (B-A) = \phi \end{cases}$$



$$\therefore P(B) = P(A) + P(B-A) \geq P(A)$$

$$\therefore P(A) \leq P(B)$$