1.1 Sets. covered in 16

read this

1.2 Probabilistic Models

A Probability Model consists of

(1) A set Ω called the sample space. The elements $\omega \in \Omega$ are called outcomes. A subset $A \subseteq \Omega$ is called an event.

(2) A <u>Probability law</u> $P(\cdot)$ that assigns to an event $A \subseteq \Omega$ a number $P(A)$ called the Probability of A.

Axioms for $P(\cdot)$

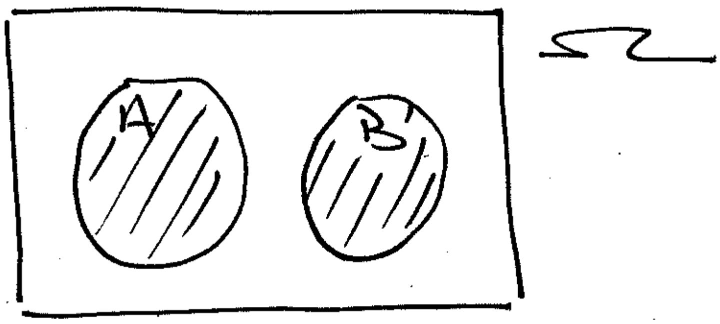
(i) $P(A) \geq 0$ for all $A \subseteq \Omega$.

(ii) $P(\Omega) = 1$

(iii) <u>Additivity</u>

- If $A, B \subseteq \Omega$ are disjoint $(A \cap B = \phi)$ then

$$P(A \cup B) = P(A) + P(B)$$



$$P(A \cup B) = P(A) + P(B)$$

"finite additivity"

- more generally, if $A_1, A_2, \ldots$ is a sequence of events $A_i \subseteq \Omega$, Pairwise disjoint ($A_i \cap A_j = \phi$ if $i \neq j$), then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

i.e.

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$ "countable additivity"

Ex. flip a fair coin

$$\Omega = \{H, T\}$$

Events: $\phi, \underbrace{\{H\}}_{H}, \underbrace{\{T\}}_{T}, \{H, T\}$

shorthand.

By axiom (iii), (ii), and $\{H\} \cap \{T\} = \phi$,
we have

$$P(H) + P(T) = P(\Omega) = 1$$

'fair' means $P(H) = P(T)$

$\therefore$ $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$.

In general, if $A \subseteq \Omega$, then

$$A \cup A^c = \Omega \text{ and } A \cap A^c = \phi$$

By Axioms (ii), (iii): $P(A) + P(A^c) = 1$

Hence by (i): $P(A) \leq 1$. Thus

$$0 \leq P(A) \leq 1$$

for any $A \subseteq \Omega$. Evidently

$$P : \mathbb{P}(\Omega) \to [0, 1].$$