CSE 107 1-15-25

<u>note</u>:

$$P(A \mid B \mid C) = \frac{P(A \cap B \mid C)}{P(B \mid C)}$$

$$= \frac{P(A \cap B \cap C) / \cancel{P(C)}}{P(B \cap C) / \cancel{P(C)}}$$

$$= \frac{P(A \cap B \cap C)}{P(B \cap C)}$$

$$= P(A \mid B \cap C)$$

Warning

$$\boxed{\begin{array}{l}\text{cond. ind.}\\ \text{of } A, B\\ \text{w.r.t. } C\end{array}} \;\begin{array}{c}\not\Rightarrow\\ \not\Leftarrow\end{array}\; \boxed{\begin{array}{l}\text{unconditional}\\ \text{ind. of } A, B\end{array}}$$

see ex. 1.20 & ex. 1.21 on p. 37

Independence of multiple events

Defn we say $A, B, C \subseteq \Omega$ are independent iff

$$(1)\quad P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$\text{Pair. ind.}\begin{cases}(2)\quad P(A \cap B) = P(A) \cdot P(B)\\ (3)\quad P(A \cap C) = P(A) \cdot P(C)\\ (4)\quad P(B \cap C) = P(B) \cdot P(C)\end{cases}$$

<u>note</u>: (2), (3), (4) together are called <u>Pairwise independence</u> of A, B, C.

<u>Warning</u>

$$\boxed{\begin{array}{c}\text{Pairwise ind.}\\ (2),(3),(4)\end{array}} \;\begin{array}{c}\not\Rightarrow\\ \not\Leftarrow\end{array}\; \boxed{\text{Property (1)}}$$

see ex. 1.22, ex. 1.23 P. 39

Binomial Probabilities

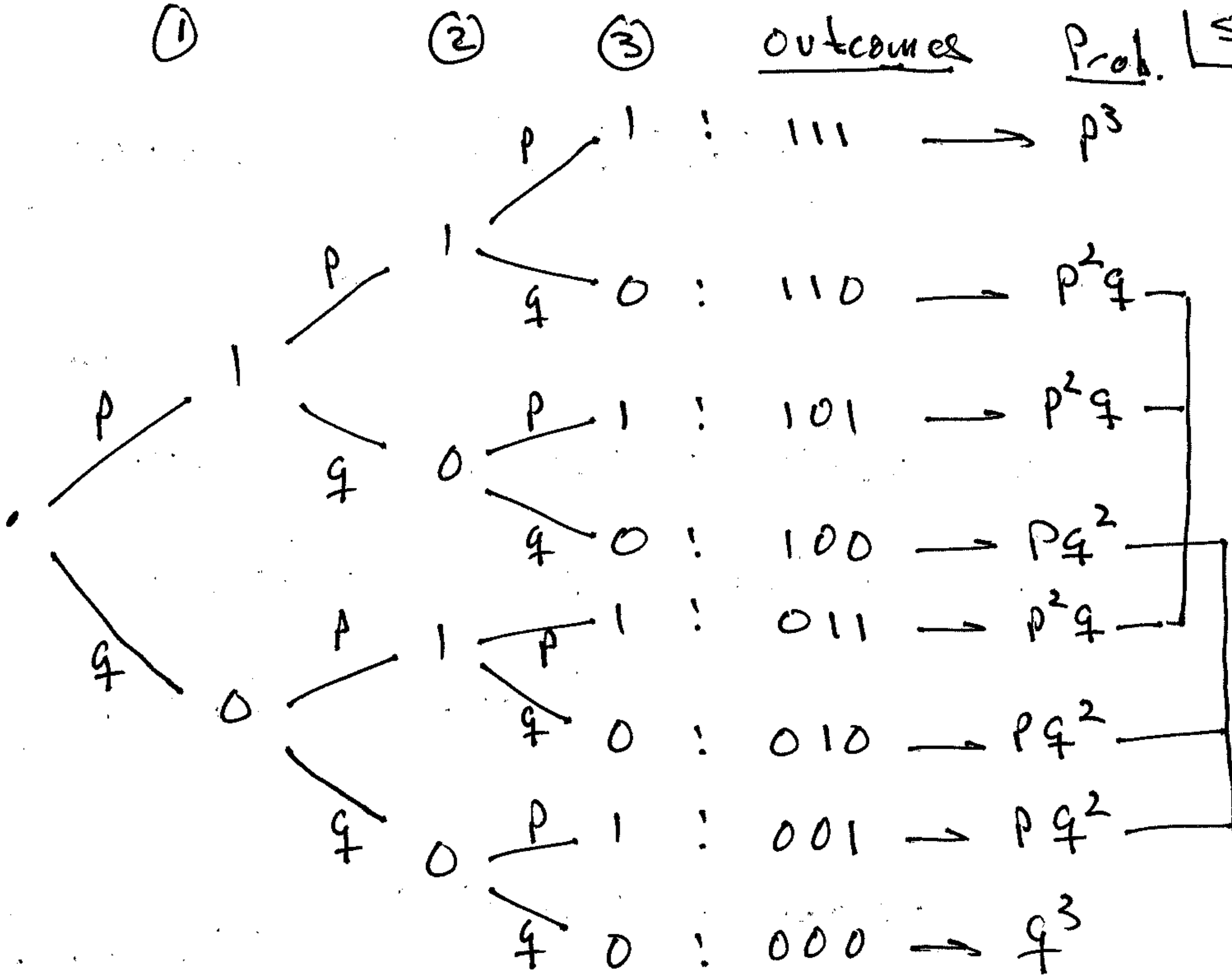
Defn

A simple experiment with only 2 outcomes is called a **Bernoulli Trial**.

$$\Omega = \begin{cases} \text{success} = \text{heads} = 1 \\ \text{failure} = \text{tails} = 0 \end{cases}$$

Ex. Perform a seq. of 3 indep. Bernoulli trials with

$$P(1) = p, \quad P(0) = 1 - p = q$$



$$p^3 + 3p^2q + 3pq^2 + q^3 = (p+q)^3 = 1$$

$$P(3 \text{ successes}) = p^3 = p^3$$

$$P(2 \quad " \quad) = 3p^2q = 3p^2(1-p)$$

$$P(1 \quad " \quad) = 3pq^2 = 3p(1-p)^2$$

$$P(0 \quad " \quad) = q^3 = q^3$$

Binomial Theorem

for any $n \geq 0$:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k \cdot y^{n-k}$$

where $\binom{n}{k} = \begin{bmatrix} \text{\# } k\text{-subsets} \\ \text{of an } n\text{-set} \end{bmatrix}$ $\quad 0 \leq k \leq n$

Binomial coefficient

$$= \begin{bmatrix} \text{\# bit strings} \\ \text{of len. } n \text{ that} \\ \text{contain } k \text{ 1's} \end{bmatrix}$$

$$= \frac{n!}{k!\,(n-k)!}$$

Theorem

In general, if we do n independent Bernoulli trials where $P(\text{success}) = p$, then

$$\boxed{P(\#\text{successes} = k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

for $0 \le k \le n$.

Binomial Probabilities

1.6 Counting / Combinitorics

Read 1.6

Review CSE 16

- # Permutations of n objects = $n!$
- # k-Permutations of n objects = $\frac{n!}{(n-k)!}$
- # k-combinations of n obj =

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

note: bijection

$$\left\{\begin{array}{l}\text{k-subsets}\\ \text{of an } n\text{-set}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\text{bit strings of}\\ \text{len. } n \text{ containing}\\ \text{exactly } k \text{ 1s}\end{array}\right\}$$

Ex. $n=4$, $k=2$. $S=\{1,2,3,4\}$

bit strs		subsets
0 0 0 0	$\longleftrightarrow$	ϕ
0 0 0 1	$\longleftrightarrow$	$\{4\}$
0 0 1 0	$\longleftrightarrow$	$\{3\}$
0 0 1 1	$\longleftrightarrow$	$\{3,4\}$
0 1 0 0	$\longleftrightarrow$	$\{2\}$
0 1 0 1	$\longleftrightarrow$	$\{2,4\}$
0 1 1 0	$\longleftrightarrow$	$\{2,3\}$
0 1 1 1		
1 0 0 0		
1 0 0 1	$\longleftrightarrow$	$\{1,4\}$
1 0 1 0	$\longleftrightarrow$	$\{1,3\}$
1 0 1 1		
1 1 0 0	$\longleftrightarrow$	$\{1,2\}$
1 1 0 1		
1 1 1 0		
1 1 1 1	$\longleftrightarrow$	$\{1,2,3,4\}$