

CSE 107
Probability and Statistics for Engineers
Information for Final Exam

Mean and Variance Formulas:

$$E[aX + b] = aE[X] + b$$

$$E[a_1X_1 + a_2X_2 + \cdots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \cdots + a_nE[X_n]$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

Convolution Product:

$$(g * h)(z) = \int_{-\infty}^{\infty} g(x)h(z - x) dx$$

Random Variables:

Discrete Uniform on $[a, b] = \{a, a + 1, a + 2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \frac{1}{b - a + 1} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{\lfloor x \rfloor - a + 1}{b - a + 1} & a \leq x < b \\ 1 & x \geq b \end{cases}$$

$$E[X] = \frac{a + b}{2} \quad \text{Var}(X) = \frac{(b - a)(b - a + 1)}{12}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1 - p & \text{if } k = 0 \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1 - p)$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \leq x < n \\ 1 & x \geq n \end{cases}$$

$$E[X] = np \quad \text{Var}(X) = np(1 - p)$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1-p)^{k-1} \cdot p & \text{if } k = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1-p)^{\lfloor x \rfloor} & \text{if } x \geq 1 \end{cases}$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1-p}{p^2}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } k = 0, 1, \dots \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } x \geq 0 \end{cases}$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$

Continuous Uniform on $[a, b]$ where $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential with parameter λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

Normal with mean μ and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad F_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$E[X] = \mu \quad \text{Var}(X) = \sigma^2$$

Standard Normal ($\mu = 0$ and $\sigma = 1$)

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

$$E[X] = 0 \quad \text{Var}(X) = 1$$

Distributions Associated with Random Processes

Pascal Distribution of order k

Let Y_k be the k^{th} arrival time in a Bernoulli process with parameter p . Then $Y_k = T_1 + T_2 + \dots + T_k$ where T_i are independent geometric random variables with parameter p .

$$p_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k} \quad \text{for } t = k, k+1, k+2, \dots$$

$$E[Y_k] = \frac{k}{p} \quad \text{Var}(Y_k) = \frac{k(1-p)}{p^2}$$

Erlang Distribution of order k

Let Y_k be the k^{th} arrival time in a Poisson process with parameter λ . Then $Y_k = T_1 + T_2 + \dots + T_k$ where T_i are independent exponential random variables with parameter λ .

$$f_{Y_k}(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} \quad \text{for } t \in [0, \infty)$$

$$E[Y_k] = \frac{k}{\lambda} \quad \text{Var}(Y_k) = \frac{k}{\lambda^2}$$

Equations Associated with Markov Chains

Consider a Markov chain model with state space $S = \{1, 2, \dots, m\}$ and state transition probabilities p_{ij} for $i, j \in S$. Let X_n denote the state at time $n \geq 0$, and let $r_{ij}(n) = P(X_n = j \mid X_0 = i)$ be the n -step transition probabilities. Then $r_{ij}(n)$ satisfies the following recurrence.

$$\text{Chapman-Kolmogorov Equations: } r_{ij}(n) = \sum_{k=1}^m r_{ik}(n-1) \cdot p_{kj} \quad \text{for all } i, j \in S \text{ and } n \geq 1$$

Suppose the Markov chain consists of one aperiodic recurrent class and possibly some transient states. Let π_j denote the steady-state probabilities for $j \in S$. Then π_j satisfy

$$\text{Balance Equations: } \pi_j = \sum_{k=1}^m \pi_k \cdot p_{kj} \quad \text{for all } j \in S$$

$$\text{Normalization Equations: } 1 = \sum_{k=1}^m \pi_k$$

Standard Normal Cumulative Distribution Function

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998