

## CSE 107

### Probability and Statistics for Engineers

#### Information for Midterm 2

#### Variance, Covariance and Correlation Formulas:

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

#### Baye's Rule, Total Probability and Total Expectation Formulas:

Let  $A_1, A_2, \dots, A_n$  be events forming a partition of  $\Omega$ , let  $B \subseteq \Omega$  be an event, and let  $X: \Omega \rightarrow \mathbb{R}$  be a random variable on  $\Omega$ .

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} \quad \text{for each } i = 1, 2, \dots, n$$

$$P(B) = \sum_{i=1}^n P(A_i) \cdot P(B|A_i)$$

$$E[X] = \sum_{i=1}^n P(A_i) \cdot E[X|A_i]$$

$$E[X] = \int_{-\infty}^{\infty} f_Y(y) \cdot E[X|Y = y] dy$$

#### Convolution Product:

$$(g * h)(z) = \int_{-\infty}^{\infty} g(x)h(z - x) dx$$

#### Random Variables:

Discrete Uniform on  $[a, b] = \{a, a + 1, a + 2, \dots, b\}$ , where  $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \frac{1}{b - a + 1} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a + b}{2} \quad \text{Var}(X) = \frac{(b - a)(b - a + 2)}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{\lfloor x \rfloor - a + 1}{b - a + 1} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

Bernoulli with parameter  $p$

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1 - p & \text{if } k = 0 \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1 - p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Binomial with parameters  $n, p$

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np \quad \text{Var}(X) = np(1 - p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \leq x < n \\ 1 & x \geq n \end{cases}$$

Geometric with parameter  $p$

$$p_X(k) = \begin{cases} (1 - p)^{k-1} \cdot p & \text{if } k = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1 - p}{p^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1 - p)^{\lfloor x \rfloor} & x \geq 1 \end{cases}$$

Poisson with parameter  $\lambda$

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } k = 0, 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & x \geq 0 \end{cases}$$

Continuous Uniform on  $[a, b]$  where  $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

Exponential with parameter  $\lambda$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

Normal with mean  $\mu$  and variance  $\sigma^2$

$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \mu \quad \text{Var}(X) = \sigma^2$$

$$F_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

Standard Normal ( $\mu = 0$  and  $\sigma = 1$ )

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$E[X] = 0 \quad \text{Var}(X) = 1$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

## Standard Normal Cumulative Distribution Function

|     | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |