CSE 107
Lab Assignment 2

In this assignment you will simulate random the process described in the following exercise from the recommended text *Probability and Random Processes* by Grimmett and Stirzaker.

> An urn contains a azure balls and c carmine balls, where $a > 0$ and $c > 0$. Balls are selected from the urn at random and discarded, until the first time a selected ball has a color different from its predecessor. That ball is then replaced, and the procedure is restarted. (Observe that the first selection has no predecessor, and is therefore discarded. Therefore each iteration of this procedure results in at least one ball being discarded.) The process continues until the last ball is discarded. Show that this last ball is equally likely to be azure or carmine.

In particular, it is asserted that $P(\text{last ball is azure}) = P(\text{last ball is carmine}) = 1/2$, and that this probability is independent of the initial values of a and c. The exercise quoted above asks the reader to prove this fact. Here you will merely demonstrate the claim experimentally.

Notice that, unlike most other random processes we've studied, the probability of selecting a particular color changes after each ball is discarded. It's as if we're flipping a weighted coin that changes its probabilities after each flip.

Let the total number of balls be 100, so that $a + c = 100$, and let $a = 10, 20, 30, 40, 50$, respectively. Run 2000 trials of the experiment for each value of a, and calculate the relative frequency of the last ball discarded being azure, in each case. Notice that these relative frequencies are all close to $1/2$. Create a table formatted similarly to the one below

```
-----------------------------------------------
 #azure  #carmine   proportion ending in azure
-----------------------------------------------
   10       90         0.0000
   20       80         0.0000
   30       70         0.0000
   40       60         0.0000
   50       50         0.0000
```

where of course you replace 0.0000 by your actual relative frequencies, rounded to 4 decimal places. Try to explain why these probabilities are all the same, regardless of the initial number of azure and carmine balls. This is not to be a rigorous proof that the probability is $1/2$, but just a rational appeal to intuition.

Place this table, and your explanation, in a file called `Report.pdf`, and submit it to Gradescope before the due date. Do not submit your source code.