CSE 107
Homework Assignment 6

Section 3.4: Multiple Continuous Random Variables

1. Consider a random variable X with PDF

$$f_X(x) = \begin{cases} \frac{2x}{3} & \text{if } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and let A be the event $A = \{X \geq 1.5\}$. Determine $E[X]$, $P(A)$ and $E[X|A]$.

Section 3.5: Conditioning

2. Alice is vacationing in Monte Carlo. The amount X of money she takes to the casino each evening is a random variable with PDF of the form

$$f_X(x) = \begin{cases} ax & \text{if } 0 \leq x \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

At the end of each night of gambling, the amount Y that she takes back to her hotel room is uniformly distributed between 0 and twice the amount she came with.

   a. Find the value of a in the above formula for $f_X(x)$.
   b. Write down the conditional PDF $f_{Y|X}(y|x)$.
   c. Determine the joint PDF $f_{X,Y}(x, y)$.
   d. Determine the marginal PDF $f_Y(y)$.
   e. Find the expected value of her profit $Z = Y - X$.
   f. What is the probability that on a given night, Alice makes a positive profit at the casino?

3. Let X have a uniform distribution in the unit interval $[0, 1]$, and let Y have an exponential distribution with parameter $\lambda = 2$. Assume that X and Y are independent, and let $Z = X + Y$.

   a. Find $P(Y \geq X)$.
   b. Find the conditional PDF of Z given that $Y = y$.
   c. Find the conditional PDF of Y given that $Z = 3$.

Section 4.1: Derived Distributions

4. Let X be a random variable with PDF $f_X(x)$. Find the PDF of the random variable $Y = |X|$ in the following three cases.
   a. X is exponentially distributed with parameter λ.
   b. X is uniformly distributed in the interval $[-1, 2]$.
   c. $f_X(x)$ is a general PDF.

5. Let X and Y be independent discrete random variables with the same PMF:

$$p_X(x) = \begin{cases} 1/4 & \text{if } x = 1 \\ 1/4 & \text{if } x = 2 \\ 1/2 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

   and similarly for Y. Use the convolution product to find the PMF of $Z = X + Y$.

6. Let X be continuous uniform on the interval $[0, 2]$, and Y continuous uniform on $[3, 4]$. Assume that X and Y are independent. Use the convolution product to find the PDF of $Z = X + Y$.