

## CSE 107

### Homework Assignment 5

#### Section 3.2: Cumulative Distribution Functions

1. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Your score in test  $i$ , where  $i = 1, 2, 3$ , takes one of the values from  $i$  to 10 with equal probability  $1/(11 - i)$ , independently of the scores in other tests. What is the PMF of the final score?
2. The *median* of a random variable  $X$  is defined to be a number  $m$  satisfying  $F_X(m) = 1/2$ . Find the median of the exponential random variable with parameter  $\lambda$ .

#### Section 3.3: Normal Random Variables

3. A radar tends to overestimate the distance of an aircraft, and the error is a normal random variable with a mean of 50 meters and a standard deviation 100 meters. What is the probability that the measured distance will be smaller than the true distance?
4. Let  $X$  be normal with mean 1 and variance 4, and let  $Y = 2X + 3$ .
  - a. Calculate the PDF of  $Y$ .
  - b. Find  $P(Y \geq 0)$ .
5. A signal of amplitude  $s = 2$  is transmitted from a satellite and is corrupted by noise. The received signal is  $X = s + W$ , where the noise  $W$  is normal. When the weather is good,  $W$  has mean 0 and variance 1. When the weather is bad,  $W$  has mean 0 and variance 4. Good and bad weather are equally likely. In the absence of any weather information, determine the following.
  - a. The PDF of  $X$ .
  - b. The probability that  $X$  is between 1 and 3.
6. A transmitter sends the signals  $-1$  and  $+1$ , with probabilities  $p$  and  $1 - p$ , respectively. The communication channel introduces additive noise that is normal with mean 0 and variance  $\sigma^2$ . This noise is independent of the particular signal sent. The receiver at the other end of the channel receives a signal which is the sum of the transmitted signal and the channel noise.
  - a. Let  $a$  be a constant between  $-1$  and  $+1$ . The receiver interprets its signal as  $-1$  if it is less than  $a$ , and as  $+1$  if it is greater than  $a$ . Find a formula (in terms of  $a$ ,  $p$  and  $\sigma$ ) for the probability that the receiver makes an error.
  - b. Determine the probability that the receiver makes an error if  $p = 2/5$ ,  $a = 1/2$  and  $\sigma^2 = 1/4$ .