

## CSE 107

### Homework Assignment 3

#### Section 2.4: Expectation, Mean and Variance

1. A class with 300 students consists of 250 undergraduate students and 50 graduate students. The probability that an undergraduate student gets an A is  $1/3$ , and the probability that a graduate student gets an A is  $1/2$ . Let  $X$  be the number of students who get an A. Determine the expectation  $E[X]$ .

Hint: define random variables

$$U_i = \begin{cases} 1 & \text{if undergraduate student } i \text{ gets an A} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } 1 \leq i \leq 250$$

and

$$G_j = \begin{cases} 1 & \text{if graduate student } j \text{ gets an A} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } 1 \leq j \leq 50$$

Write  $X$  in terms of the  $U_i$  and  $G_j$ , then use linearity of expectation.

#### Section 2.5: Joint PMFs of multiple random variables

2. The UCSC football team wins any one game with probability  $p$ , and loses it with probability  $1 - p$ . Its performance in each game is independent of its performance in other games. Let  $L_1$  be the number of losses before its first win, and let  $L_2$  be the number of losses after its first win and before its second win. Find the joint PMF of  $L_1$  and  $L_2$ .
3. A class of  $n$  students takes a test in which each student gets an A with probability  $p$ , a B with probability  $q$ , and a grade below B with probability  $1 - p - q$ , independently of any other student. Let  $X$  and  $Y$  be the numbers of students that getting A's and a B's, respectively. Determine the joint PMF  $p_{X,Y}(x, y)$ .

#### Section 2.6: Conditioning

4. Alvin shops for probability books for  $K$  hours, where  $K$  is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books  $N$  that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N|K}(n|k) = \begin{cases} \frac{1}{k} & \text{if } n = 1, \dots, k \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the joint PMF of  $K$  and  $N$ .
- (b) Find the marginal PMF of  $N$ .
- (c) Find the conditional PMF of  $K$  given that  $N = 2$ .