CSE 107
Homework Assignment 2

Section 1.4: Total probability and Bayes rule

1. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 85% accurate if the student is in fact overstressed. It is known that 99.5% of all students are overstressed. Given that a particular student tests negative for stress, what is the probability that the test results are correct, and that this student is not overstressed?

Section 1.5: Independence

2. Three persons each independently roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13} and A_{23} are pairwise independent but are not independent.

Section 2.2: Probability mass functions

3. The annual premium of a special kind of insurance starts at $1000 and is reduced by 10% after each year where no claim has been filed. The probability that a claim is filed in a given year is 0.05, independently of preceding years. What is the PMF of the total premium paid up to and including the year when the first claim is filed?

Section 2.3: Functions of a random variable

4. Let X be a discrete random variable that is uniformly distributed over the set of integers in the range $[a, b]$, where a and b are integers with $a < 0 < b$. Find the PMF of the random variables $\max(0, X)$ and $\min(0, X)$.

5. Let X be a discrete random variable, and let $Y = |X|$.

   (a) Assume that the PMF of X is

   $$p_X(x) = \begin{cases} cx^2 & \text{if } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

   where c is a suitable constant. Determine the value of c.

   (b) For the PMF of X given in part (a) calculate the PMF of Y.

   (c) Give a general formula for the PMF of $Y = |X|$ in terms of the PMF of X.

Section 2.4: Expectation, mean and variance

6. Let X be a random variable that takes integer values and is symmetric, that is,

$$P(X = k) = P(X = -k)$$

for all integers k. What is the expected value of $Y = cos(X\pi)$ and $Y = sin(X\pi)$?

7. A particular binary data transmission and reception device is prone to some error when receiving data. Suppose that each bit is read correctly with probability p. Find the smallest value of p such that when 10,000 bits are received, the expected number of errors is at most 10.

8. Let N be a nonnegative integer-valued random variable. Show that

$$E[N] = \sum_{i=1}^{\infty} P(N \geq i)$$