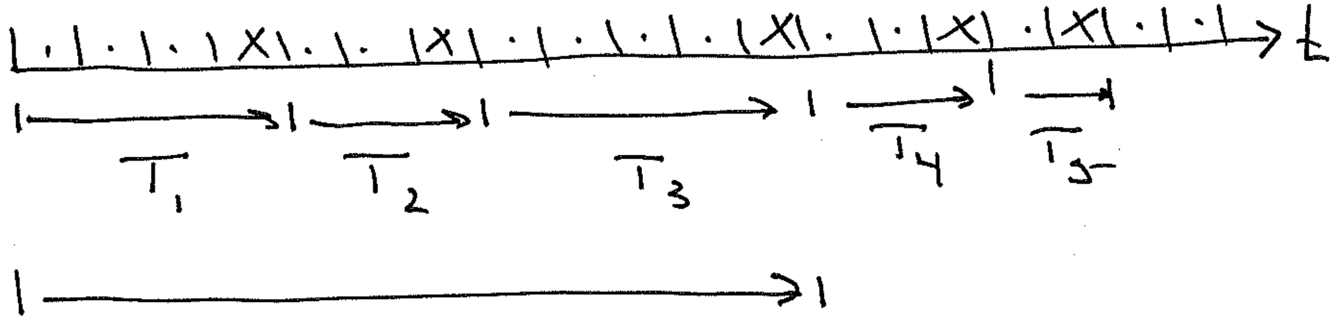
CSE 107 2-29-24

Picture of Bernoulli Process



$$Y_3 = T_1 + T_2 + T_3$$

In general: $Y_K = T_1 + \cdots + T_K$

Also could define:

$$T_1 = Y_1$$

$$T_K = Y_K - Y_{K-1} \quad (K = 2, 3, \ldots)$$

We see: T_k are independent

T_k is geometric(p)

(PMF)

What is the distribution of Y_k?

One way: $Y_k = T_k + Y_{k-1}$

compute this recursively using

convolution product.

<u>Exercise</u>: do this.

shortcut: use independence

$$P_{Y_K}(t) = P(Y_K = t)$$

$$= P\big(\underbrace{(K-1)\text{ arrivals in }[1,t-1]}\text{ and }\underbrace{\text{arrival at }t}\big)$$

$$= \underbrace{P\big((K-1)\text{ arrivals in }[1,t-1]\big)}_{\text{Binomial}(p,\,t-1,\,K-1)} \cdot \underbrace{P(1\text{ arrival at }t)}_{p}$$

$$= \binom{t-1}{K-1} \cdot p^{K-1}(1-p)^{(t-1)-(K-1)} \cdot p$$

so

$$P_{Y_K}(t) = \begin{cases} \binom{t-1}{K-1} p^K (1-p)^{t-K} & t = K, K+1, \dots \\ 0 & \text{otherwise} \end{cases}$$

Called Pascal PMF.