CSE 107
Probability and Statistics for Engineers
Some Common Random Variables

1. Discrete Uniform on $[a, b] = \{a, a+1, a+2, \ldots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \dfrac{1}{b-a+1} & \text{if } a \le k \le b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)(b-a+2)}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{\lfloor x \rfloor - a + 1}{b-a+1} & a \le x < b \\ 1 & x \ge b \end{cases}$$

2. Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1-p & \text{if } k = 0 \end{cases}$$

$$E[X] = p$$

$$Var(X) = p(1-p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

3. Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{if } 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np$$

$$Var(X) = np(1-p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1-p)^{n-k} & 0 \le x < n \\ 1 & x \ge n \end{cases}$$

4. Geometric with parameter p

$$p_X(k) = \begin{cases} (1-p)^{k-1} \cdot p & \text{if } k = 1, 2, 3, \dots \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p}$$

$$Var(X) = \frac{1-p}{p^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1-p)^{\lfloor x \rfloor} & x \geq 1 \end{cases}$$

5. Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \dfrac{\lambda^k}{k!} & \text{if } k = 0, 1, 2, 3, \dots \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \lambda$$

$$Var(X) = \lambda$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & x \geq 0 \end{cases}$$

6. Continuous Uniform on $[a, b]$ where $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & a \leq x < b \\ 1 & x \geq b \end{cases}$$

7. Exponential with parameter λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

8. Normal with mean μ and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \mu$$

$$Var(X) = \sigma^2$$

$$F_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}}\, dt$$

9. Standard Normal ($\mu = 0$ and $\sigma = 1$)

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$E[X] = 0$$

$$Var(X) = 1$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt$$

Distributions Associated with Random Processes

<u>Pascal Distribution of order k</u>
Let Y_k be the k^{th} arrival time in a Bernoulli process with rate p. Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent geometric random variables with paramenter p.

$$p_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k} \qquad \text{for } t = k, k+1, k+2, \ldots$$

$$E[Y_k] = \frac{k}{p}$$

$$Var(Y_k) = \frac{k(1-p)}{p^2}$$

<u>Erlang Distribution of order k</u>
Let Y_k be the k^{th} arrival time in a Poisson process with rate λ. Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent exponential random variables with paramenter λ.

$$f_{Y_k}(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} \qquad \text{for } t \in [0, \infty)$$

$$E[Y_k] = \frac{k}{\lambda}$$

$$Var(Y_k) = \frac{k}{\lambda^2}$$