CSE 107
Lab Assignment 1

In this assignment you will simulate the experiment described in Problem 27 on page 59 of the textbook, which says:

> Alice and Bob have $2n + 1$ coins, each with probability of a head equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining n coins. Assuming independent coin tosses, show that the probability that after all the coins have been tossed, Bob will have gotten more heads than Alice, is $1/2$.

Write a program, in any language, that simulates this experiment with n equal to 300. Perform 10,000 trials of this experiment, and compute the relative frequency of Bob tossing more heads than Alice.

$$\text{relative frequency} = \frac{\text{number of trials in which Bob tossed more heads}}{\text{total number of trials}}$$

Note this relative frequency is an approximation to $P(\text{Bob tosses heads more than Alice})$. Verify that your relative frequency is very close to $1/2$.

Now suppose that we do another sequence of 10,000 trials with $2n + 1$ *loaded* coins (again $n = 300$). In particular, run all 10,000 trials of the experiment, but now with the probability of heads equal to p. Do this for each

$$p \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$$

This should be possible by editing just a few lines of code (replacing $1/2$ by a variable p which you initialize at the beginning of the program.) Does the probability that Bob tosses more heads than Alice now seem to depend on p, or is it still $1/2$? Form a conjecture regarding the probability that Bob tosses more heads than Alice, and whether this probability depends (strongly, weakly, or not at all) on p. Create a table containing, for each value of $p \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$, the relative frequency with which Bob throws more heads than Alice. Again you are running each experiment, with $n = 300$ and doing 10,000 trials, for each value of p. Your table should be formatted similarly to the one below,

```
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 p      relative frequency
-------------------------
 0.2    0.0000
 0.3    0.0000
 0.4    0.0000
 0.5    0.0000
 0.6    0.0000
 0.7    0.0000
 0.8    0.0000
```

where of course you replace `0.0000` with your actual relative frequencies, rounded to 4 decimal places. Once your table is constructed, state your conjecture as to the probability that Bob tosses more heads, and how (strongly, weakly, or not at all) that probability depends on p. Submit your answers in a pdf file (called `lab1.pdf`) to Gradescope before the due date.