CSE 107
Homework Assignment 4

Section 2.7: Independence

1. Joe Lucky plays the lottery on any given week with probability p, independently of whether he played on any other week. Each time he plays, he has a probability q of winning, again independently of everything else. During a fixed time period of n weeks, let X be the number of weeks that he played the lottery and Y be the number of weeks that he won.

    a. What is the probability that he played the lottery on any particular week, given that he did not win on that week?

    b. Find the conditional PMF $p_{Y|X}(y|x)$.

    c. Find the joint PMF $p_{X,Y}(x, y)$.

    d. Find the marginal PMF $p_Y(y)$. Hint: The standard approach would be to take the answer to (c) and sum over x. This will give the correct answer, but with much algebra. Instead, try to see why Y is a binomial random variable. What are its parameters?

    e. Find the conditional PMF $p_{X|Y}(x|y)$. Do this algebraically using the preceding answers.

2. Ike the Spike performs 10 independent rolls a single fair 6-sided die. Let X be the sum of the outcomes of all 10 rolls. Find the $E[X]$ and $\mathrm{Var}(X)$. (Hint: let X_i be the number on top of the die on the i^{th} roll ($1 \leq i \leq 10$), and observe $X = X_1 + X_2 + \cdots + X_{10}$.)

Section 3.1: Continuous Random Variables and PDFs

3. The runner-up in a road race is given a reward that depends on the difference between his time and the winner's time. He is given 10 dollars for being at most 1 minute behind, 6 dollars for being more than 1 and at most 3 minutes behind, 2 dollars for being more than 3 and at most 6 minutes behind, and nothing otherwise. Given that the difference between his time and the winner's time is uniformly distributed between 0 and 12 minutes, find the mean and variance of the reward of the runner-up.

4. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} cx & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

    and let $Y = X^3$.

    a. Determine the value c.
    b. Find $E[Y]$.
    c. Find $\mathrm{Var}(Y)$ and σ_Y.

5. Let Θ be a continuous uniform random variable on the interval $0 \leq \theta \leq \pi/2$, and let $X = \cos(\Theta)$. Find $E[X]$ and $\mathrm{Var}(X)$.

Section 3.2: Cumulative Distribution Functions

6. Find the PDF, the mean, and the variance of the random variable X with CDF

$$F_X(x) = \begin{cases} 1 - \dfrac{a^3}{x^3} & \text{if } x \geq a \\ 0 & \text{if } x < a \end{cases}$$

where a is a positive constant.

7. The *median* of a random variable X is defined to be a number m satisfying $F_X(m) = 1/2$. Find the median of the exponential random variable with parameter λ.

8. Let $0 < p < 1$, and let X be the continuous random variable with PDF

$$f_X(x) = \begin{cases} pe^x & \text{if } x < 0 \\ (1-p)e^{-x} & \text{if } x \geq 0 \end{cases}$$

   a. Find the mean $E[X]$ of X.

   b. Find the variance $\mathrm{Var}(X)$ of X.

   c. Find the CDF $F_X(x)$ of X.

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