

CSE 107

Homework Assignment 1

Section 1.2:

1. We roll a fair 4-sided die 3 times, and let the results be $r_1, r_2, r_3 \in \{1, 2, 3, 4\}$, respectively. Assume each outcome is equally likely.
 - (a) How many different outcomes (r_1, r_2, r_3) are there to this experiment?
 - (b) Find the probability that r_1 is even.
 - (c) Find the probability that $r_2 < r_3$.
 - (d) Find the probability that $r_1 < r_2 < r_3$.
2. Alice and Bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:
 - A: The magnitude of the difference of the two numbers is greater than $1/3$.
 - B: At least one of the numbers is greater than $1/3$.
 - C: The two numbers are equal.
 - D: Alice's number is greater than $1/3$.

Find the probabilities

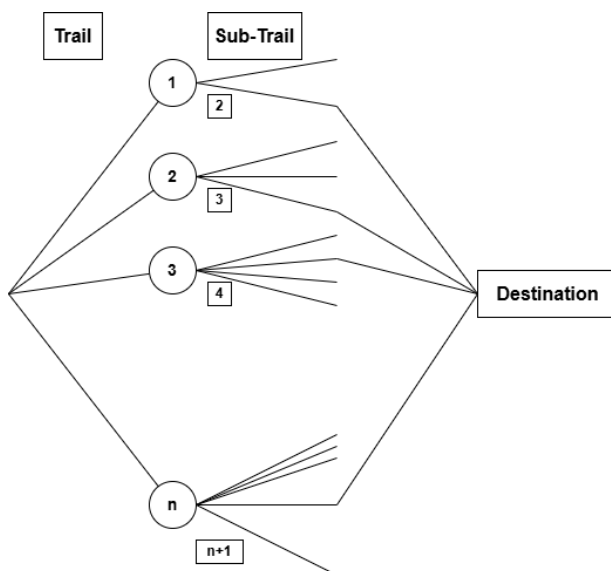
- (a) $P(A)$
- (b) $P(B)$
- (c) $P(A \cap B)$
- (d) $P(C)$
- (e) $P(D)$
- (f) $P(A \cap D)$

Section 1.3:

3. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.
 - (a) Find the probability that doubles were rolled.
 - (b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.
 - (c) Find the probability that at least one die is a 6.
 - (d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.
4. A coin is tossed twice. It is claimed that the probability of getting two heads, given that the first toss is a head, is greater than or equal to the probability of getting two heads, given that at least one of the tosses is a head. Determine whether or not this claim is true. [Hint: Let A be the event that the first toss is a head, and B the event that the second toss is a head. Compare the conditional probabilities $P(A \cap B | A)$ and $P(A \cap B | A \cup B)$.]

Section 1.4:

5. A magnetic tape storing information in binary form has been corrupted, so it can only be read with bit errors. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?
6. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 85% accurate if the student is in fact overstressed. It is known that 99.5% of all students are overstressed. Given that a particular student tests negative for stress, what is the probability that the test results are correct, and that this student is not overstressed?
7. A hiker starts by taking one of n available trails, denoted $1, 2, \dots, n$. An hour into the hike, trail i subdivides into $i + 1$ subtrails, exactly one of which leads to the hiker's destination. The hiker has no map and makes random choices of trail and subtrail. Find a formula, as a function of n , giving the probability of reaching the destination.



8. Urn A contains 4 white and 3 black balls, and Urn B contains 3 white and 7 black balls. An urn is selected at random, and ball is picked at random from it.
 - (a) What is the probability that the ball is black?
 - (b) If the ball is white, what is the probability that Urn B was selected?