cse 107 5-28-26

Exercise

If $A, B \subseteq \Omega$ are indep., and $P(A) > 0$, then A, B are conditionally indep. given A. i.e.

$$P(A \cap B \mid A) = P(A \mid A) \cdot P(B \mid A)$$

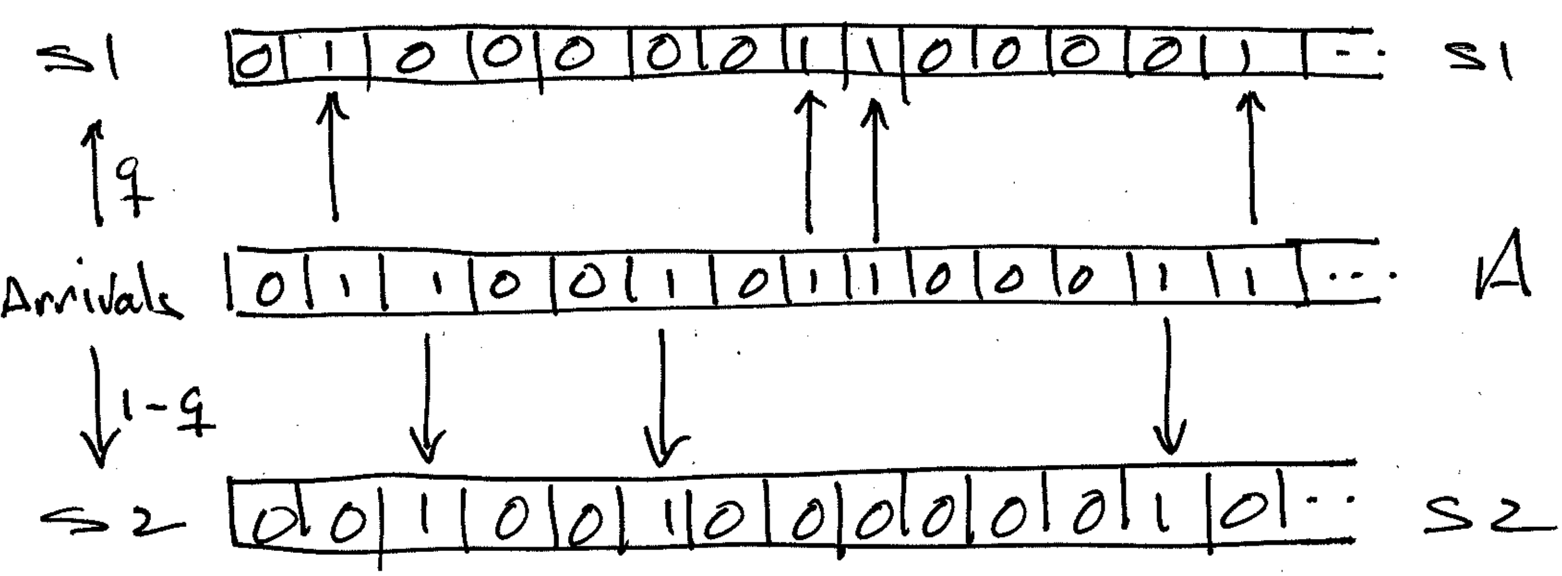
Splitting a Bernoulli Process

Say we have a B.P. of Parameter (rate) p Producing a stream of arrivals. Each Job goes to one of 2 servers with

$$P(S1) = q$$

$$P(S2) = 1 - q$$

note: coin deciding S1 or S2 is indep. of coin giving arrival stream.



What kind of Process is seen by S1?

" " " " " " " S2

- Rate of S1 is pq B.P.
- Rate of S2 is $p(1-q)$ B.P.

Merging Bernoulli Processes

Have 2 $\overset{\text{indep.}}{\wedge}$ Simultaneous B.P.s

rate

S1 : $X_1, X_2, \ldots\ldots$ p

S2 : $Y_1, Y_2, \ldots$ q

Merged stream

rate = $p+q-pq$

M : $M_1, M_2, \ldots\ldots$

inclusive or

Rule

$$M_t = \begin{cases} 1 & \text{if either } X_t=1 \text{ or } Y_t=1 \\ 0 & \text{otherwise} \end{cases}$$

What kind of Process is M? Observe

$$P(M_t=1) = 1 - P(M_t=0)$$

$$= 1 - P(X_t=0, Y_t=0)$$

$$= 1 - P(X_t=0)\cdot P(Y_t=0)$$

$$= 1 - (1-p)(1-q)$$

$$= 1 - (1 - p - q + pq)$$

$$= p + q - pq$$

Exercise

- 1st split, then merge (using or). What's the rate of the resulting B.P.
- 1st merge, then split. What's the rates of resulting B.P.
- Change bit operation to: and, exor, nand, nor, implies. find rate of Merge operation.

Summary: B.P.

- $\{X_t \mid t = 1, 2, 3, \ldots\}$
  - indep.
  - $P(X_t = 1) = \underline{\underline{p}}$ for all t.
    rate
- Y_k = time to k^{th} arrival
- $\begin{cases} T_1 = Y_1 \\ T_k = Y_k - Y_{k-1} \quad (k = 2, 3 \ldots) \end{cases}$
- $N_t = X_1 + X_2 + \cdots + X_t$
  = # arrivals in $\{1, 2, \ldots, t\}$

<u>note</u>: $N_t \sim \text{Binomial}(t, p)$

$$\therefore \quad P_{N_t}(k) = P(N_t = k)$$

$$= \binom{t}{k} p^k (1-p)^{t-k} \quad (k = 0, 1, \ldots, t)$$

<u>note</u>

$$\begin{cases} X_1 = N_1 \\ X_t = N_t - N_{t-1} \quad (t = 2, 3, \ldots \end{cases}$$

<u>also</u>

$$Y_k = \min\{t \mid N_t = k\}$$

6.2 The Poisson Process

Defn

An Arrival Process is a set of r.v.s

$$\{X_t \mid t \in S\}$$

where $S \subseteq \mathbb{R}$.

$$X_t = \begin{cases} 1 & \text{if arrival at time } t \\ 0 & \text{if no arrival " " "} \end{cases}$$

- Discrete time: $S \subseteq \mathbb{R}$ is a discrete subset.

- Continuous time:

$S = \mathbb{R}^+ = [0, \infty)$, and

$\{t \mid X_t = 1\}$ is a discrete subset of $\mathbb{R}^+$.

<u>Defn</u>

an arrival Process is <u>time homogeneous</u> iff

$$P(k \text{ arrivals in } [t_0, t_0 + t])$$

is independent of t_0

<u>Notation</u>

$$h(k,t) = P(k \text{ arrivals in } [t_0, t_0 + t])$$

Defn

a continuous time arrival Process is called a Poisson Process iff

(a) time homogeniety

This is like each trial has same Prob. of success in ~~xxx~~ a B.P.

(b) independence

the # of arrivals in disjoint intervals are independent.

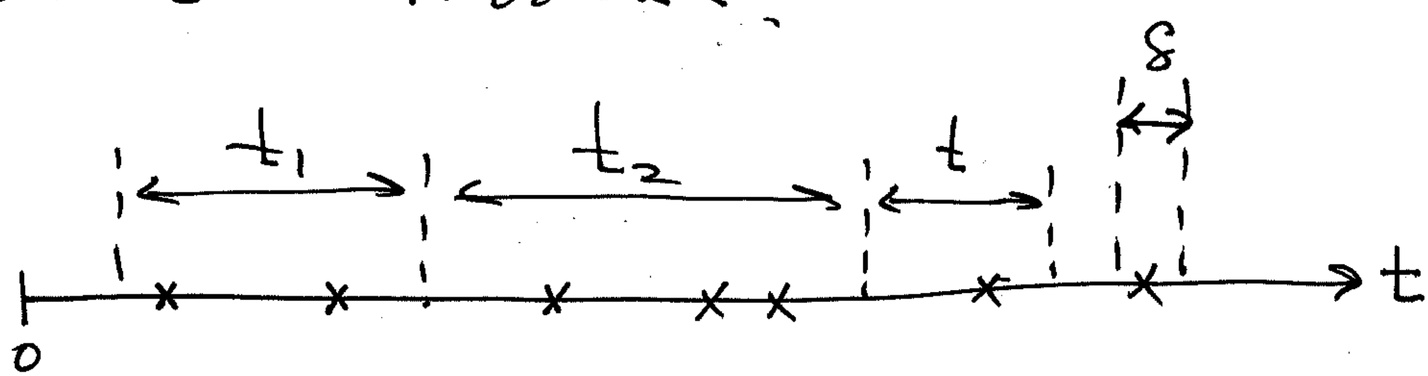
this is like different trials are indep. in a B.P.

(c) Small interval Probabilities

Let $\delta > 0$ be small. Then

$$h(k,\delta) \approx \begin{cases} 1-\lambda\delta & \text{if } k=0 \\ \lambda\delta & \text{if } k=1 \\ 0 & \text{if } k=2,3,4,\ldots \end{cases}$$

where λ is the 'arrival rate' of the Process.



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