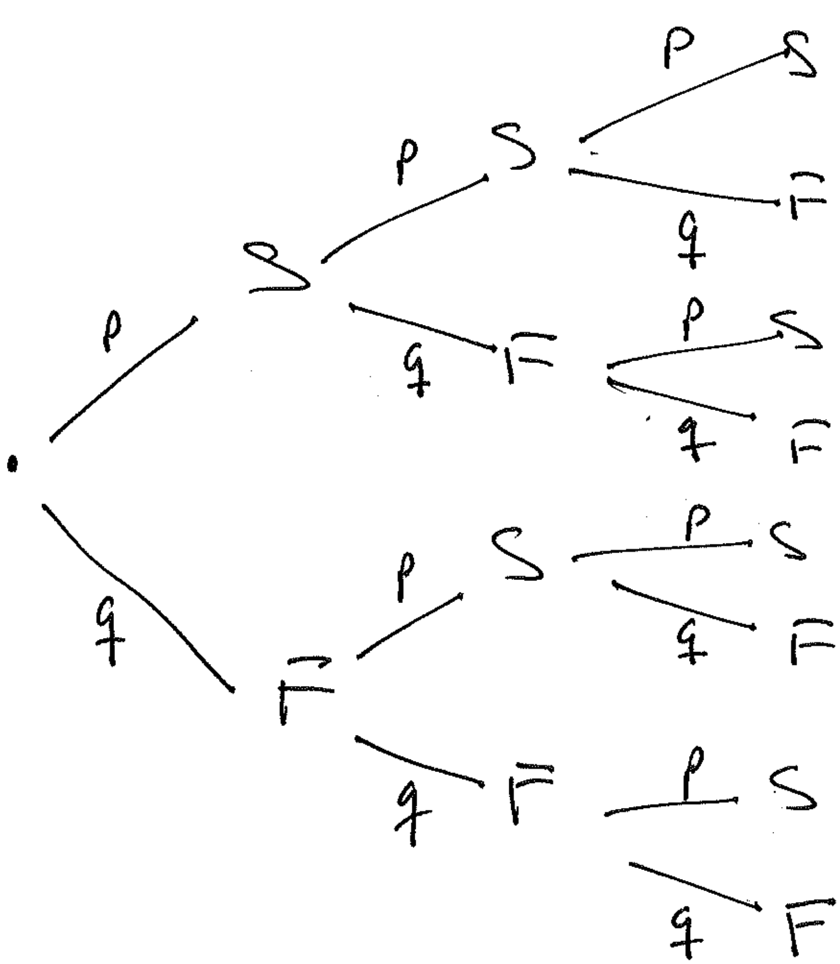
L1

cse 107 4-9-26

Ex. Perform 3 independent Bernoulli trials with

$$P(\text{success}) = p$$

$$P(\text{failure}) = 1 - p = q$$



SSS	p^3
SSF	$p^2 q$
SFS	$p^2 q$
SFF	$p q^2$
FSS	$p^2 q$
FSF	$p q^2$
FFS	$p q^2$
FFF	q^3

Check

$$p^3 + 3p^2 q + 3pq^2 + q^3 = (p+q)^3 = 1^3 = 1 \checkmark$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \underbrace{\binom{n}{k}}_{\text{Binomial coefficient}} x^k y^{n-k}$$

					1					$n=0$
				1		1				$n=1$
			1		2		1			$n=2$
		1		3		3		1		$n=3$
	1		4		6		4		1	$\vdots$
$\vdots$		$\vdots$			$\vdots$			$\ddots$		$\ddots$

$$\binom{4}{2} = \frac{4!}{2!\,2!}$$

Recall

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad (0 \le k \le n)$$

$$= \left[\begin{array}{l}\text{\# of } k\text{-element} \\ \text{subsets of an } n\text{-} \\ \text{element set}\end{array}\right]$$

$$= \left[\begin{array}{l}\text{\# of bit strings of} \\ \text{length } n \text{ having exactly} \\ k \text{ 1's.}\end{array}\right]$$

In general, if we do n independent Bernoulli trials $P(\text{success}) = p$, then

$$\boxed{P(\#\text{successes} = k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

Binomial Probabilities

where $0 \le k \le n$

1.6 Combinatorics

Read 1.6

- Permutations of n objects

$$\boxed{\#\text{Perm} = n!}$$

- k-Permutations of n objects $(0 \le k \le n)$.

$$\boxed{\#\,k\text{-Perm} = n(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}}$$

- k-combinations of n objects

$$\binom{n}{k} = \boxed{\begin{array}{l}\#\,k\text{-ssets of}\\ \text{an } n\text{-set}\end{array}}$$

$$= \boxed{\begin{array}{l}\#\text{ bit strings of len. } n\\ \text{having } k \text{ 1's}\end{array}}$$

Ex $n = 4$, $\boxed{k = 2}$

$S = \{1, 2, 3, 4\}$ Subset

$$0\ 0\ 0\ 0 \longleftrightarrow \{\ \}$$
$$0\ 0\ 0\ 1 \longleftrightarrow \{4\}$$
$$0\ 0\ 1\ 0 \longleftrightarrow \{3\}$$
$$\boxed{0\ 0\ 1\ 1 \longleftrightarrow \{3, 4\}}$$
$$0\ 1\ 0\ 0 \longleftrightarrow \{2\}$$
$$\boxed{0\ 1\ 0\ 1 \longleftrightarrow \{2, 4\}}$$
$$\boxed{0\ 1\ 1\ 0 \longleftrightarrow \{2, 3\}}$$
$$0\ 1\ 1\ 1 \longleftrightarrow \{2, 3, 4\}$$
$$1\ 0\ 0\ 0 \longleftrightarrow \{1\}$$
$$\boxed{1\ 0\ 0\ 1 \longleftrightarrow \{1, 4\}}$$
$$\boxed{1\ 0\ 1\ 0 \longleftrightarrow \{1, 3\}}$$
$$1\ 0\ 1\ 1 \longleftrightarrow \{1, 3, 4\}$$
$$\boxed{1\ 1\ 0\ 0 \longleftrightarrow \{1, 2\}}$$
$$1\ 1\ 0\ 1 \longleftrightarrow \{1, 2, 4\}$$
$$1\ 1\ 1\ 0 \longleftrightarrow \{1, 2, 3\}$$
$$1\ 1\ 1\ 1 \longleftrightarrow \{1, 2, 3, 4\}$$

- Partitions of n objects into k sets with set i having n_i objects ($1 \le i \le n$ and $n_1 + \dots + n_k = n$)

$$\underbrace{\binom{n}{n_1, n_2, \dots, n_k}}_{\text{multinomial coefficients}} = \frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

Ex. # anagrams of MISSISSIPPI

$$\left.\begin{array}{l} n_1 = \#M = 1 \\ n_2 = \#I = 4 \\ n_3 = \#S = 4 \\ n_4 = \#P = 2 \end{array}\right\} \#\text{anagrams} = \frac{11!}{1! \cdot 4! \cdot 4! \cdot 2!}$$

- k-combinations of n objects where repetition is allowed.

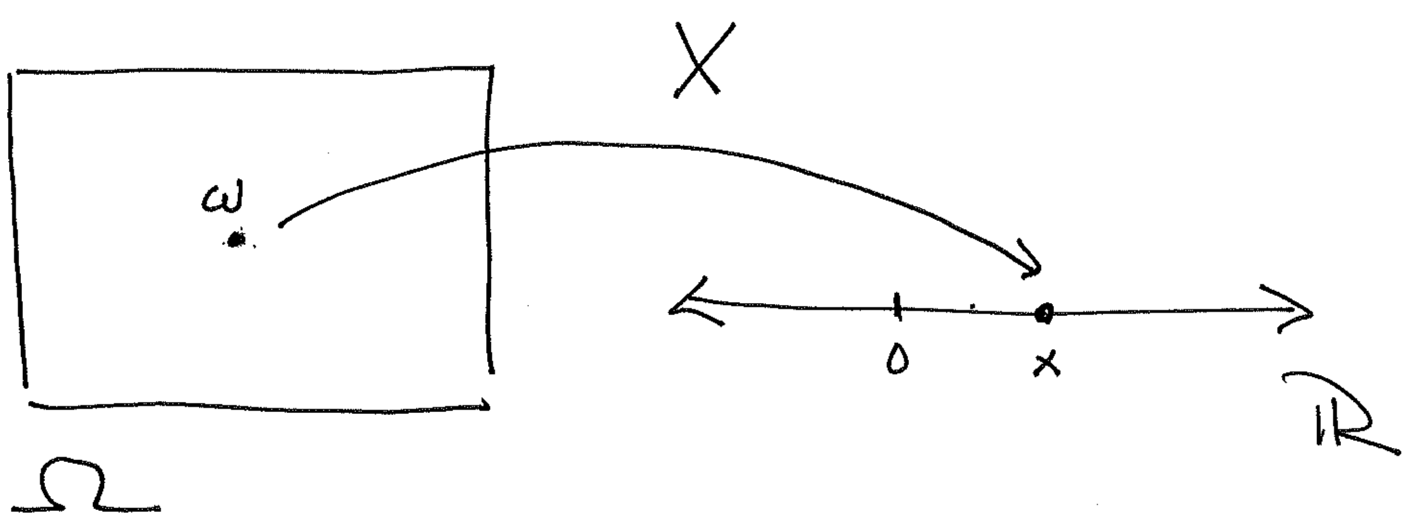
$$\boxed{\begin{array}{c}\#\ k\text{-comb.}\\ \text{with rep}\end{array}} = \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

2.1 Random Variables

Defn

A random variable is a function

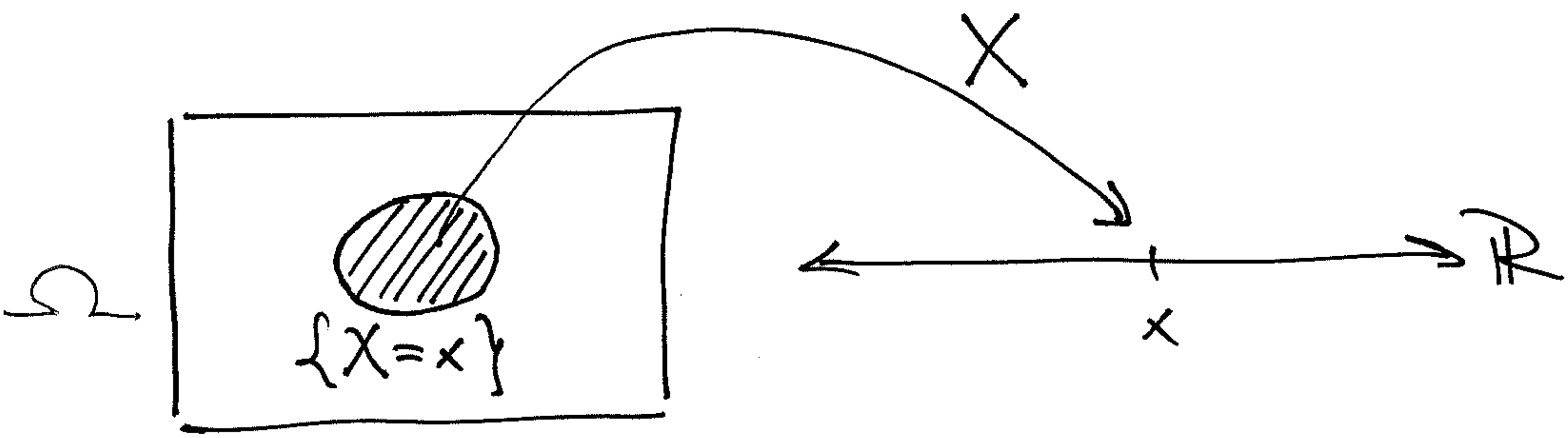
$$X : \Omega \to \mathbb{R}.$$



notation: $X(\omega) = x$

- we denote by $\{X=x\}$ the event

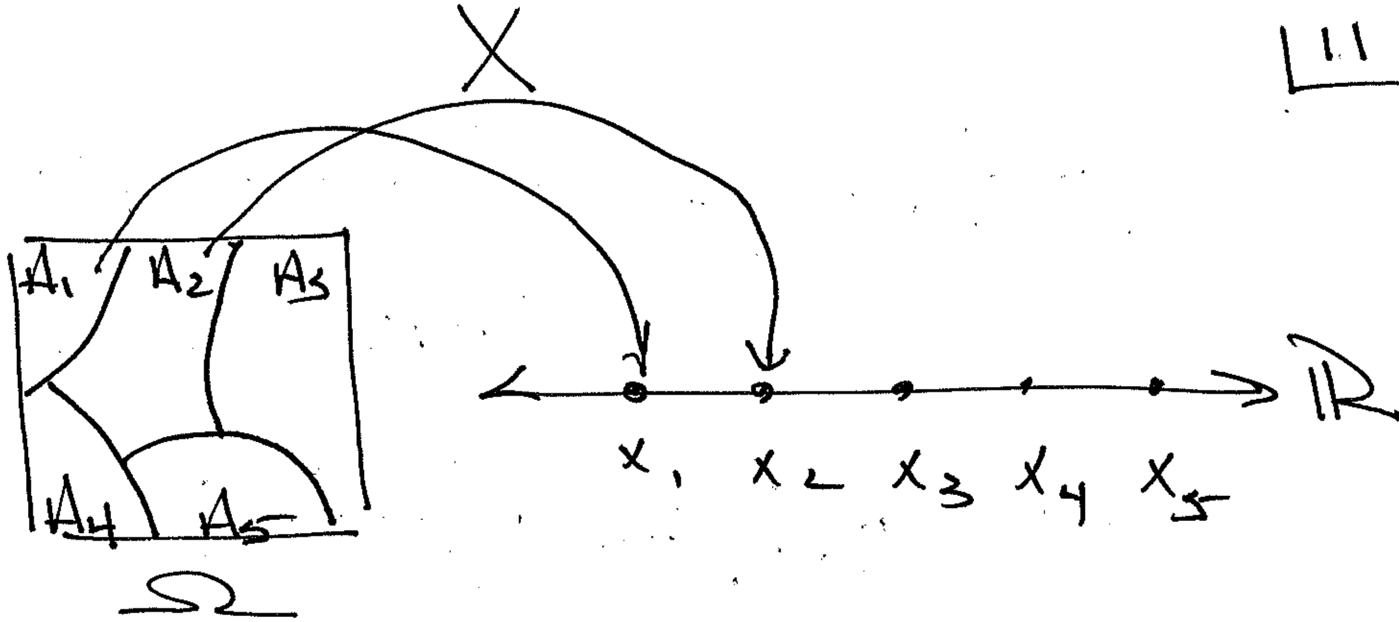
$$\{X=x\}=\{\omega\in\Omega \mid X(\omega)=x\}$$



- X is said to be <u>discrete</u> iff its range

$$\mathrm{range}(X)=\{X(\omega) \mid \omega\in\Omega\}\subseteq\mathbb{R}$$

is a discrete subset of $\mathbb{R}$,



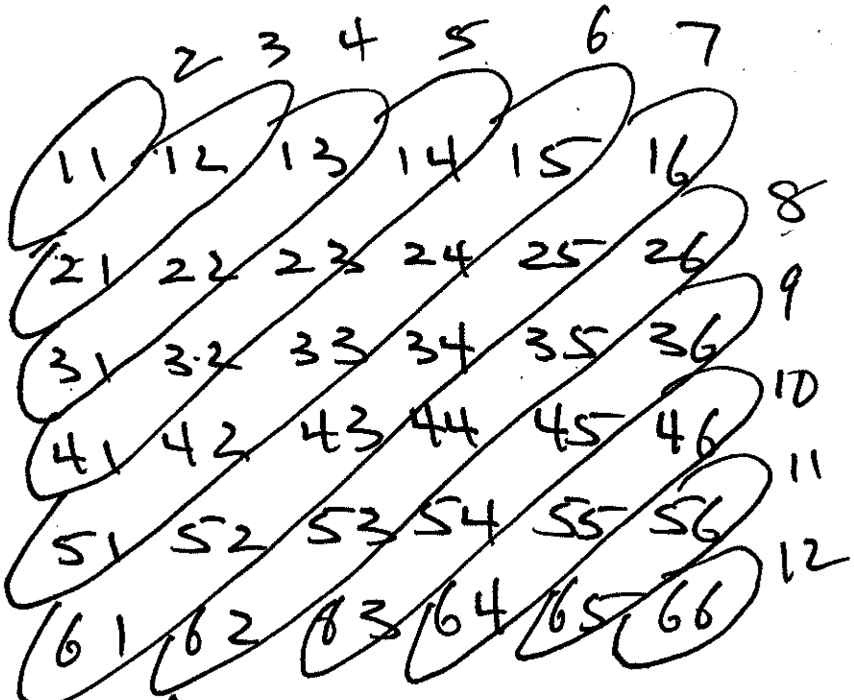
$$A_i = \{X = x_i\}$$

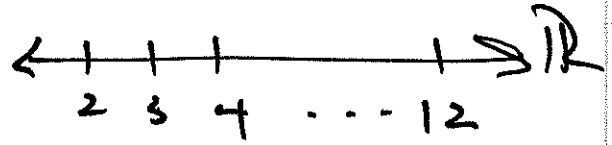
- X is continuous . . . .

Ex.

Roll 2 fair indep. dice. Let

$$X((i,j)) = i+j$$





range(X)

$= \{2,3,4,5,6,7,8,9,10,11,12\}$

$\{X=8\} = \{(i,j) \mid i+j=8\}$

2.2 Probability mass functions

Defn

Let X be a discrete r.v.

The *Probability mass function* (PMF) of X is

$$p_X(x) = P(\{X = x\}) = P(X = x)$$

Observe $p_X : \mathbb{R} \to [0, 1]$.

Ex. (Previous) $p_X(8) = \frac{5}{36}$