CSE 107 4-14-26

- Bernoulli(p)
- Binomial(n, p)
- Geometric(p)
- Poisson(λ)
- Uniform(a, b)

2.4 Expected Value, mean, Variance

Defn

The expected value (or mean) of a discrete r.v. X is

$$\boxed{E[X] = \sum_{x} x\, p_X(x)}$$

sum over all $x \in$ range(X)

<u>Ex.</u> (Previous)

$$P_X(k) = \begin{cases} .2 & \text{if } k=-1 \\ .3 & \text{if } k=1 \\ .5 & \text{if } k=2 \\ 0 & \text{otherwise} \end{cases}$$

$$Y = X^2$$

$$P_Y(k) = \begin{cases} .5 & \text{if } k=1 \\ .5 & \text{if } k=4 \\ 0 & \text{otherwise} \end{cases}$$

so

$$E[X] = (-1)(.2) + 1\cdot(.3) + 2(.5)$$

$$= \dots = \boxed{1.1}$$

$$E[Y] = E[X^2]$$

$$= 1\cdot(.5) + 4\cdot(.5)$$

$$= \boxed{2.5}$$

Expected Value rule

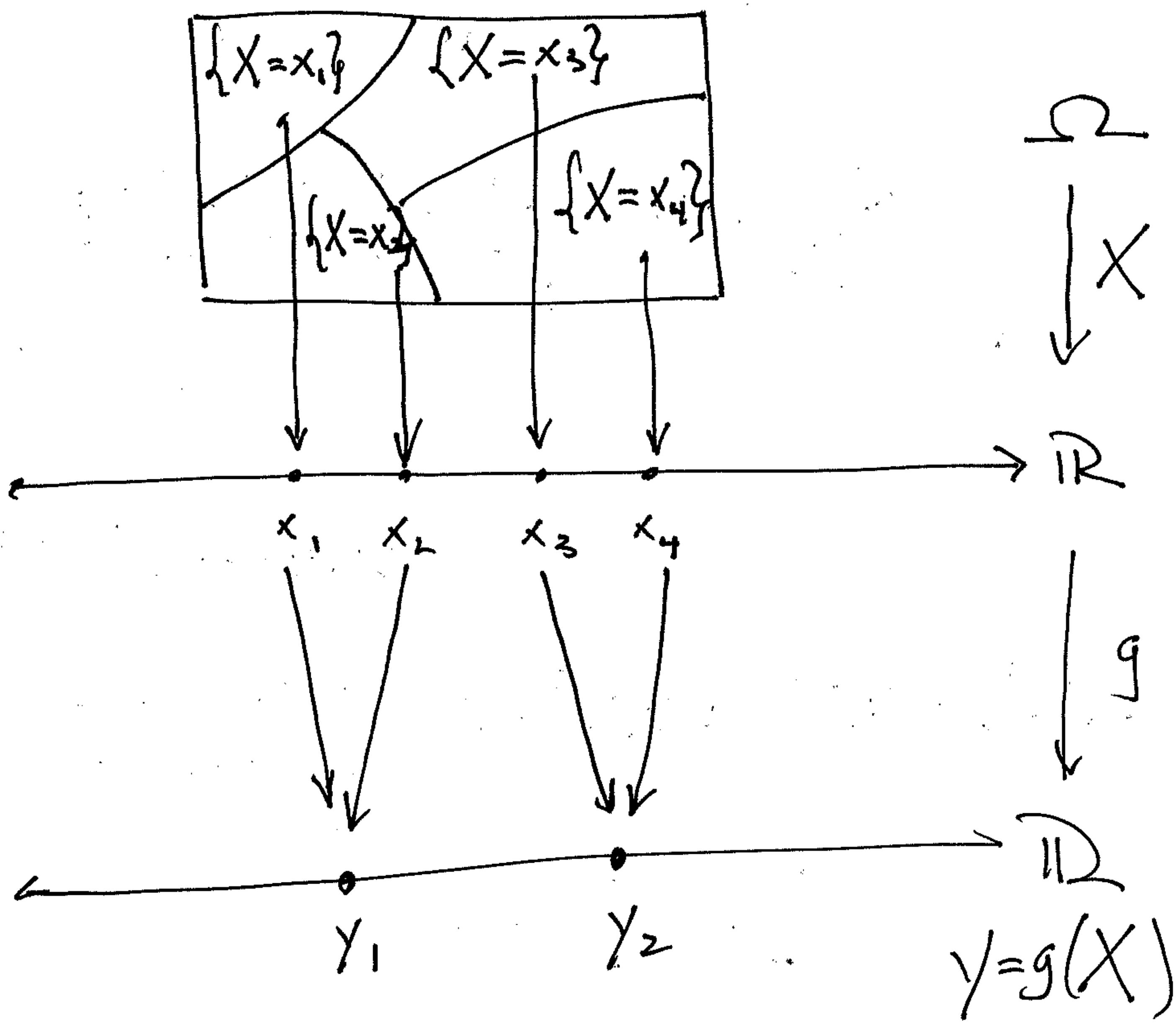
$$\boxed{E[g(X)] = \sum_x g(x) \cdot p_X(x)}$$

Ex. (Previous)

$$E[X^2] = (-1)^2 \cdot (.2) + 1^2 \cdot (.3) + 2^2 \cdot (.5)$$

$$= \ldots = \boxed{2.5}$$

Picture:



$$\mathbb{E}[Y] = y_1 \cdot p_Y(y_1) + y_2\, p_Y(y_2)$$

$$= y_1\big(p_X(x_1) + p_X(x_2)\big) + y_2\big(p_X(x_3) + p_X(x_4)\big)$$

$$= y_1 p_X(x_1) + y_1 p_X(x_2) + y_2 p_X(x_3) + y_2 p_X(x_4)$$

$$= g(x_1) p_X(x_1) + g(x_2) \cdot p_X(x_2) + g(x_3) \cdot p_X(x_3) + g(x_4) \cdot p_X(x_4)$$

$$= \sum_{\substack{x \\ x \in \{x_1, x_2, x_3, x_4\}}} g(x) p_X(x)$$

<u>Defn</u>

The <u>n^{th} moment</u> of X

$(n = 0, 1, 2, 3, \ldots)$ is

$$E[X^n]$$

Thus mean $E[X]$ is 1^{st} moment.

Defn

The variance of X is the mean the r.v. $(X-E[X])^2$:

$$\boxed{Var(X) = E\left[(X-E[X])^2\right]}$$

note: $(X-E[X])^2 \geq 0$, so $Var(X) \geq 0$

note:

$Var(X)$ is a measure of how 'spread-out' mass is about mean $E[X]$

Defn

The standard deviation of X is

$$\sigma_X = \sqrt{Var(X)}$$

note: σ_X has same units as X.

Ex. (previous)

$$p_X(k) = \begin{cases} .2 & \text{if } k=-1 \\ .3 & k=1 \\ .5 & k=2 \\ 0 & \text{otherwise} \end{cases}$$

Recall

$$E[X] = 1.1 \qquad (1^{st} \text{ moment})$$

$$E[X^2] = 2.5 \qquad (2^{nd} \text{ moment})$$

$$Var(X) = E[(X-E[X])^2]$$

$$= \sum_x (x-(1.1))^2 p_X(x)$$

$$= (-1-(1.1))^2 \cdot (.2) + (1-(1.1))^2(.3) + (2-(1.1))^2(.5)$$

$$= \cdots = \boxed{1.29}$$

and

$$\sigma_X = \sqrt{1.29} = \boxed{1.1358}$$