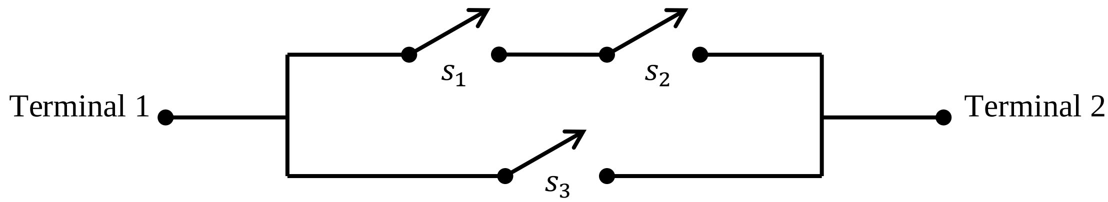
CSE 107
Midterm 1 Review Problems

1. In the circuit diagram below, switches s_1, s_2 and s_3 are randomly and independently set in the open or closed state. Let A_i be the event that s_i is **open** (for $i = 1, 2, 3$), and let B be the event that there is a **closed path** from terminal 1 to terminal 2.



   Suppose that $P(A_i) = p_i$, for $i = 1, 2, 3$. Determine $P(B)$ in terms of p_1, p_2 and p_3.

2. Two independent rolls of a fair 6-sided die are performed, and the number on top (1, 2, 3, 4, 5 or 6) is observed. Define the events A, B and C as:

$$A = \{\text{the first roll is } 2, 3 \text{ or } 4\}$$
$$B = \{\text{the second roll is } 3 \text{ or } 4\}$$
$$C = \{\text{the sum is } 8\}$$

   (Hint: circle the above events in the sample space pictured below.)

11	**12**	**13**	**14**	**15**	**16**
21	**22**	**23**	**24**	**25**	**26**
31	**32**	**33**	**34**	**35**	**36**
41	**42**	**43**	**44**	**45**	**46**
51	**52**	**53**	**54**	**55**	**56**
61	**62**	**63**	**64**	**65**	**66**

   a. Are events A and B *independent*? Justify your answer.

   b. Are events A and B *conditionally independent given* C? Justify your answer.

3. A system consists of n identical components, each of which is operational with probability p, independent of other components. The system is operational if at least m out of the n components are operational. What is the probability that the system is operational?

4. The UCSC football team will play 16 games this season. Each game ends in a win, loss or tie. Let W be the number of wins, and L be the number of losses. Assume that the outcome of each game is independent of the outcomes of other games, and that UCSC wins/loses/ties with fixed probabilities p, q and r (respectively) in each game.

   a. If $E[W] = 3.2$, what is the probability p with which UCSC wins? (Hint: think of there as being two possible outcomes: $\{\text{win}, \text{not win}\}$).

   b. If $E[L] = 11.2$, what is the probability q with which UCSC loses? (Hint: again, think of there being two possible outcomes: $\{\text{lose}, \text{not lose}\}$).

   c. What is the expected number of games until the first tie?

   d. What is the probability that there are no ties?

5. Alice and Bob have a chess match in which the first player to win a game wins the match. Each game has one of 3 possible outcomes: Bob wins, Alice wins, or the game is a draw. One game is played each day until someone wins, so the match is of potentially unlimited duration. The prize money starts at \$100 and goes up by \$100 each day a match is played. Alice wins with probability 0.4, Bob wins with probability 0.3, and a draw occurs with probability 0.3.

   a. What is the probability that Alice wins the match?

   b. Determine the mean and standard deviation of the total prize money.

6. A transmitter sends bits 0 and 1 through a noisy channel, where $P(1\text{ is transmitted}) = 0.6$. A detector at the other end receives these bits with errors. If a 0 is transmitted, then a 0 is detected with probability 0.9. If a 1 is transmitted, then a 1 is detected with probability 0.85. Let T_0 be the event that a 0 was transmitted, T_1 the event that a 1 was transmitted, D_0 the event that a 0 was detected, and D_1 the event that a 1 was detected.

   a. Find $P(D_0|T_0)$, $P(D_1|T_1)$ and $P(T_1)$ directly from the given information.

   b. Deduce $P(D_1|T_0)$, $P(D_0|T_1)$ and $P(T_0)$ from your answers in part (a).

   c. Use Bayes rule to determine $P(T_1|D_1)$.

   d. Use Bayes rule to determine $P(T_0|D_0)$.

7. A 3-sided die and a coin, which are neither fair nor independent, are rolled and tossed, respectively. The die has faces $\{1, 2, 3\}$ and the coin has sides labeled $\{1, 2\}$. Let X be the outcome of the die, and Y the outcome of the coin. The *conditional* PMF $p_{X|Y}(x|y)$ is given by the following table.

y	1	$2/8$	$5/8$	$1/8$
	2	$1/8$	$3/8$	$4/8$
		1	2	3
			x	

Also, the *marginal* PMF $p_Y(y)$ is given by the following table.

y	1	$1/3$
	2	$2/3$

a. Fill in the following table giving the *joint* PMF $p_{X,Y}(x, y)$.

y	1			
	2			
		1	2	3
			x	

b. Fill in the following table giving the *marginal* PMF $p_X(x)$.

1	2	3
	x	

c. Fill in the following table giving the conditional PMF $p_{Y|X}(y|x)$.

y	1			
	2			
		1	2	3
			x	

d. Given that the coin flip is 2, what is the probability that the die roll is 3?

8. Let X and Y be discrete random variables with $\text{range}(X) = \text{range}(Y) = \{1, 2, 3\}$. Suppose the joint PMF $p_{X,Y}(x, y)$ is given by the following table.

y \ x	1	2	3
3	0.15	0.05	0.10
2	0.20	0.10	0.05
1	0.05	0.25	0.05

a. Determine the marginal PMFs $p_X(x)$ and $p_Y(y)$, and fill in the following tables.

$p_X(x)$:

x	1	2	3

$p_Y(y)$:

y	
3	
2	
1	

b. Determine the conditional PMF $p_{X|Y}(x|y)$, and fill in the following table.

$p_{X|Y}(x|y)$:

y \ x	1	2	3
3			
2			
1			

c. Determine the conditional PMF $p_{Y|X}(y|x)$, and fill in the following table.

$p_{Y|X}(y|x)$:

y \ x	1	2	3
3			
2			
1			

d. Determine the conditional expectation $E[X \mid Y = 2]$.

9. The number X of phone calls received by a call center within a certain time period is a Poisson random variable with parameter λ. Determine the smallest positive number λ such that the probability of receiving at least one call is at least $1/2$.

10. We perform independent rolls of a fair 6-sided die until the number on top is at least 5. Let the random variable X be the number of rolls performed.

    a. Determine the PMF of X.

    b. Determine the mean and variance of X.

    c. Determine $P(X \leq 2)$.