

CSE 107

Probability and Statistics for Engineers

Information for Midterm 2

Variance, Covariance and Correlation Formulas:

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

Baye's Rule, Total Probability and Total Expectation Formulas:

Let $A_1, A_2, \dots, A_n$ be events forming a partition of Ω , let $B \subseteq \Omega$ be an event, and let $X: \Omega \rightarrow \mathbb{R}$ be a random variable on Ω .

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} \quad \text{for each } i = 1, 2, \dots, n$$

$$P(B) = \sum_{i=1}^n P(A_i) \cdot P(B|A_i)$$

$$E[X] = \sum_{i=1}^n P(A_i) \cdot E[X|A_i]$$

$$E[X] = \int_{-\infty}^{\infty} f_Y(y) \cdot E[X|Y = y] dy$$

Convolution Product:

$$(g * h)(z) = \int_{-\infty}^{\infty} g(x)h(z - x) dx$$

Random Variables:

Discrete Uniform on $[a, b] = \{a, a + 1, a + 2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \frac{1}{b - a + 1} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a + b}{2} \quad \text{Var}(X) = \frac{(b - a)(b - a + 2)}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{\lfloor x \rfloor - a + 1}{b - a + 1} & a \leq x < b \\ 1 & x \geq b \end{cases}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1 - p & \text{if } k = 0 \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1 - p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np \quad \text{Var}(X) = np(1 - p)$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \leq x < n \\ 1 & x \geq n \end{cases}$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1 - p)^{k-1} \cdot p & \text{if } k = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1 - p}{p^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1 - p)^{\lfloor x \rfloor} & x \geq 1 \end{cases}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } k = 0, 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & x \geq 0 \end{cases}$$

Continuous Uniform on $[a, b]$ where $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

Exponential with parameter λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

Normal with mean μ and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \mu \quad \text{Var}(X) = \sigma^2$$

$$F_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

Standard Normal ($\mu = 0$ and $\sigma = 1$)

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$E[X] = 0 \quad \text{Var}(X) = 1$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

Standard Normal Cumulative Distribution Function

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998