

CSE 107
Probability and Statistics for Engineers
Information for Midterm 1

Mean and Variance Formulas:

$$E[aX + b] = aE[X] + b$$

$$E[a_1X_1 + a_2X_2 + \cdots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \cdots + a_nE[X_n]$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

Baye's Rule, Total Probability and Total Expectation Formulas:

Let $A_1, A_2, \dots, A_n$ be events forming a partition of Ω , let $B \subseteq \Omega$ be an event, and let $X: \Omega \rightarrow \mathbb{R}$ be a random variable on Ω .

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} \quad \text{for each } i = 1, 2, \dots, n$$

$$P(B) = \sum_{i=1}^n P(A_i) \cdot P(B|A_i) = \sum_{i=1}^n P(A_i \cap B)$$

$$E[X] = \sum_{i=1}^n P(A_i) \cdot E[X|A_i]$$

Summation Formulas:

$$\sum_{k=0}^{n-1} ar^k = a \left(\frac{1-r^n}{1-r} \right) \quad \text{for any } r \neq 1$$

$$\sum_{k=0}^{\infty} ar^k = a \left(\frac{1}{1-r} \right) \quad \text{for } -1 < r < 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \quad \text{for any } x \in \mathbb{R}$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

Discrete Random Variables:

Discrete Uniform on $\{a, a + 1, a + 2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \frac{1}{b - a + 1} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a + b}{2} \quad \text{Var}(X) = \frac{(b - a)(b - a + 2)}{12}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1 - p & \text{if } k = 0 \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1 - p)$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np \quad \text{Var}(X) = np(1 - p)$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1 - p)^{k-1} \cdot p & \text{if } k = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1 - p}{p^2}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } k = 0, 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$