CSE 107
Homework Assignment 5

Section 3.2: Cumulative Distribution Functions

1. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Your score in test i, where $i = 1, 2, 3$, takes one of the values from i to 10 with equal probability $1/(11 - i)$, independently of the scores in other tests. What is the PMF of the final score?

Section 3.3: Normal Random Variables

2. A city's high temperature on a given date is modeled as a normal random variable with mean and standard deviation both equal to 10 degrees Celsius. What is the probability that the high temperature on that date will be less than or equal to 59 degrees Fahrenheit? (Note: the conversion from Celsius to Fahrenheit is $F = (9/5)C + 32$.)

3. A radar tends to overestimate the distance of an aircraft, and the error is a normal random variable with a mean of 50 meters and a standard deviation 100 meters. What is the probability that the measured distance will be smaller than the true distance?

4. Let X be normal with mean 1 and variance 4, and let $Y = 2X + 3$.

   a. Calculate the PDF of Y.
   b. Find $P(Y \geq 0)$.

5. A signal of amplitude $s = 2$ is transmitted from a satellite and is corrupted by noise. The received signal is $X = s + W$, where the noise W is normal. When the weather is good, W has mean 0 and variance 1. When the weather is bad, W has mean 0 and variance 4. Good and bad weather are equally likely. In the absence of any weather information, determine the following.

   a. The PDF of X.
   b. The probability that X is between 1 and 3.

6. A transmitter sends the signals -1 and $+1$, with probabilities p and $1 - p$, respectively. The communication channel introduces additive noise that is normal with mean 0 and variance σ^2. This noise is independent of the particular signal sent. The receiver at the other end of the channel receives a signal which is the sum of the transmitted signal and the channel noise.

   a. Let a be a constant between -1 and $+1$. The receiver interpretes its signal as -1 if it is less than a, and as $+1$ if it is greater than a. Find a formula (in terms of a, p and σ) for the probability that the receiver makes an error.
   b. Determine the probability that the receiver makes an error if $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.

Section 3.4: Joint PDFs of Multiple Continuous Random Variables

7. A point is chosen at random (according to a uniform PDF) within a semicircle S of the form

$$S = \{(x, y) | x^2 + y^2 \leq r^2, y \geq 0\}$$

for some $r > 0$.

a. Find the joint PDF $f_{X,Y}(x, y)$ of the coordinates X and Y of the chosen point.
b. Find the marginal PDF $f_Y(y)$ and use it to find $E[Y]$.
c. Find $E[Y]$ directly (without using $f_Y(y)$) by using the formula $E[Y] = \iint_S y f_{X,Y}(x, y)\, dx\, dy$. Hint: use polar coordinates to compute this integral.

Section 3.5: Conditioning

8. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} \dfrac{c}{x^2} & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Determine the constant c.
b. Let $A = \{X > 5/4\}$. Find $P(A)$ and determine the conditional PDF $f_{X|A}(x)$.
c. Find the conditional expectation $E[X|A]$.
d. Find the conditional second moment $E[X^2|A]$ and the conditional variance $\mathrm{Var}(X|A)$.