

CSE 107
Homework Assignment 4

Section 2.7: Independence

1. Joe Lucky plays the lottery on any given week with probability p , independently of whether he played on any other week. Each time he plays, he has a probability q of winning, again independently of everything else. During a fixed time period of n weeks, let X be the number of weeks that he played the lottery and Y be the number of weeks that he won.
 - a. What is the probability that he played the lottery on any particular week, given that he did not win on that week?
 - b. Find the conditional PMF $p_{Y|X}(y|x)$.
 - c. Find the joint PMF $p_{X,Y}(x,y)$.
 - d. Find the marginal PMF $p_Y(y)$. Hint: The standard approach would be to take the answer to (c) and sum over x . This will give the correct answer, but with much algebra. Instead, try to see why Y is a binomial random variable. What are its parameters?
 - e. Find the conditional PMF $p_{X|Y}(x|y)$. Do this algebraically using the preceding answers.
2. Ike the Spike performs 10 independent rolls a single fair 6-sided die. Let X be the sum of the outcomes of all 10 rolls. Find the $E[X]$ and $\text{Var}(X)$. (Hint: let X_i be the number on top of the die on the i^{th} roll ($1 \leq i \leq 10$), and observe $X = X_1 + X_2 + \cdots + X_{10}$.)

Section 3.1: Continuous Random Variables and PDFs

3. The runner-up in a road race is given a reward that depends on the difference between his time and the winner's time. He is given 10 dollars for being one minute behind, 6 dollars for being one to three minutes behind, 2 dollars for being 3 to 6 minutes behind, and nothing otherwise. Given that the difference between his time and the winner's time is uniformly distributed between 0 and 12 minutes, find the mean and variance of the reward of the runner-up.
4. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} \frac{2x}{3} & \text{if } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and let $Y = X^2$. Calculate $E[Y]$ and $\text{Var}(Y)$.

5. Let Θ be a continuous uniform random variable on the interval $0 \leq \theta \leq \pi/2$, and let $X = \cos(\theta)$. Find $E[X]$ and $\text{Var}(X)$.

Section 3.2: Cumulative Distribution Functions

6. Find the PDF, the mean, and the variance of the random variable X with CDF

$$F_X(x) = \begin{cases} 1 - \frac{a^3}{x^3} & \text{if } x \geq a \\ 0 & \text{if } x < a \end{cases}$$

where a is a positive constant.

7. The *median* of a random variable X is defined to be a number m satisfying $F_X(m) = 1/2$. Find the median of the exponential random variable with parameter λ .

8. Let $0 < p < 1$, and let X be the continuous random variable with PDF

$$f_X(x) = \begin{cases} pe^x & \text{if } x < 0 \\ (1-p)e^{-x} & \text{if } x \geq 0 \end{cases}$$

- Find the mean $E[X]$ of X .
- Find the variance $\text{Var}(X)$ of X .
- Find the CDF $F_X(x)$ of X .