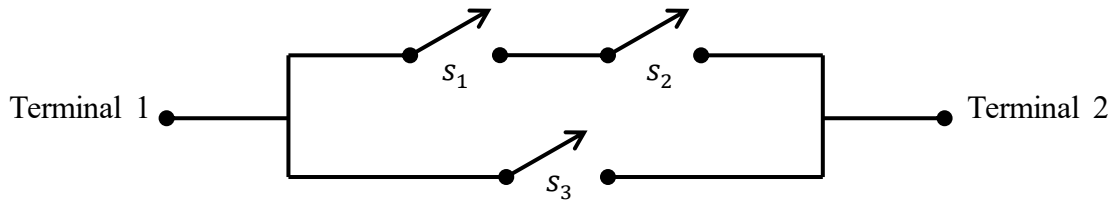


CSE 107

Midterm 1 Review Problems

1. In the circuit diagram below, switches s_1 , s_2 and s_3 are randomly and independently set in the open or closed state. Let A_i be the event that s_i is **open** (for $i = 1, 2, 3$), and let B be the event that there is a **closed path** from terminal 1 to terminal 2.



Suppose that $P(A_i) = p_i$, for $i = 1, 2, 3$. Determine $P(B)$ in terms of p_1 , p_2 and p_3 .

2. Two independent rolls of a fair 6-sided die are performed, and the number on top (1, 2, 3, 4, 5 or 6) is observed. Define the events A , B and C as:

$$A = \{\text{the first roll is 2, 3 or 4}\}$$

$$B = \{\text{the second roll is 3 or 4}\}$$

$$C = \{\text{the sum is 8}\}$$

(Hint: circle the above events in the sample space pictured below.)

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

a. Are events A and B *independent*? Justify your answer.

b. Are events A and B *conditionally independent given C* ? Justify your answer.

3. A system consists of n identical components, each of which is operational with probability p , independent of other components. The system is operational if at least m out of the n components are operational. What is the probability that the system is operational?
4. The UCSC football team will play 16 games this season. Each game ends in a win, loss or tie. Let W be the number of wins, and L be the number of losses. Assume that the outcome of each game is independent of the outcomes of other games, and that UCSC wins/loses/ties with fixed probabilities p , q and r (respectively) in each game.
- If $E[W] = 3.2$, what is the probability p with which UCSC wins? (Hint: think of there as being two possible outcomes: {win, not win}).
 - If $E[L] = 11.2$, what is the probability q with which UCSC loses? (Hint: again, think of there being two possible outcomes: {lose, not lose}).
 - What is the expected number of games until the first tie?
 - What is the probability that there are no ties?

5. Alice and Bob have a chess match in which the first player to win a game wins the match. Each game has one of 3 possible outcomes: Bob wins, Alice wins, or the game is a draw. One game is played each day until someone wins, so the match is of potentially unlimited duration. The prize money starts at \$100 and goes up by \$100 each day a match is played. Alice wins with probability 0.4, Bob wins with probability 0.3, and a draw occurs with probability 0.3.
- What is the probability that Alice wins the match?
 - Determine the mean and standard deviation of the total prize money.
6. A transmitter sends bits 0 and 1 through a noisy channel, where $P(1 \text{ is transmitted}) = 0.6$. A detector at the other end receives these bits with errors. If a 0 is transmitted, then a 0 is detected with probability 0.9. If a 1 is transmitted, then a 1 is detected with probability 0.85. Let T_0 be the event that a 0 was transmitted, T_1 the event that a 1 was transmitted, D_0 the event that a 0 was detected, and D_1 the event that a 1 was detected.
- Find $P(D_0|T_0)$, $P(D_1|T_1)$ and $P(T_1)$ directly from the given information.
 - Deduce $P(D_1|T_0)$, $P(D_0|T_1)$ and $P(T_0)$ from your answers in part (a).
 - Use Bayes rule to determine $P(T_1|D_1)$.
 - Use Bayes rule to determine $P(T_0|D_0)$.

7. A 3-sided die and a coin, which are neither fair nor independent, are rolled and tossed, respectively. The die has faces $\{1, 2, 3\}$ and the coin has sides labeled $\{1, 2\}$. Let X be the outcome of the die, and Y the outcome of the coin. The *conditional* PMF $p_{X|Y}(x|y)$ is given by the following table.

y	1	$2/8$	$5/8$	$1/8$
	2	$1/8$	$3/8$	$4/8$
		1	2	3
		x		

Also, the *marginal* PMF $p_Y(y)$ is given by the following table.

y	1	$1/3$
	2	$2/3$

- a. Fill in the following table giving the *joint* PMF $p_{X,Y}(x,y)$.

y	1			
	2			
		1	2	3
		x		

- b. Fill in the following table giving the *marginal* PMF $p_X(x)$.

	1	2	3
	x		

- c. Fill in the following table giving the conditional PMF $p_{Y|X}(y|x)$.

y	1			
	2			
		1	2	3
		x		

- d. Given that the coin flip is 2, what is the probability that the die roll is 3?

8. Let X and Y be discrete random variables with $\text{range}(X) = \text{range}(Y) = \{1, 2, 3\}$. Suppose the joint PMF $p_{X,Y}(x,y)$ is given by the following table.

y	3	0.15	0.05	0.10
	2	0.20	0.10	0.05
	1	0.05	0.25	0.05
		1	2	3
		x		

- a. Determine the marginal PMFs $p_X(x)$ and $p_Y(y)$, and fill in the following tables.

$p_X(x) :$

	1	2	3
	x		

$p_Y(y) :$

y	3	
	2	
	1	

- b. Determine the conditional PMF $p_{X|Y}(x|y)$, and fill in the following table.

$p_{X|Y}(x|y) :$

y	3			
	2			
	1			
		1	2	3
		x		

- c. Determine the conditional PMF $p_{Y|X}(y|x)$, and fill in the following table.

$p_{Y|X}(y|x) :$

y	3			
	2			
	1			
		1	2	3
		x		

- d. Determine the conditional expectation $E[X | Y = 2]$.

9. The number X of phone calls received by a call center within a certain time period is a Poisson random variable with parameter λ . Determine the smallest positive number λ such that the probability of receiving at least one call is at least $1/2$.
10. We perform independent rolls of a fair 6-sided die until the number on top is at least 5. Let the random variable X be the number of rolls performed.
- Determine the PMF of X .
 - Determine the mean and variance of X .
 - Determine $P(X \leq 2)$.