CSE 107 9-25-25

1.1 <u>Sets</u>: covered in CSE 16.

1.2 Probabalistic Models

A <u>Probability Model</u> consists of:

(1) a set Ω called the <u>sample space</u>. elements $\omega \in \Omega$ are called <u>outcomes</u>, of some experiment. A subset $E \subseteq \Omega$ is called an <u>event</u>.

(2) A <u>Probability law</u> $\mathbb{P}$ that assigns to an event $A \subseteq \Omega$ a real number $\mathbb{P}(A)$ called the probability of A.

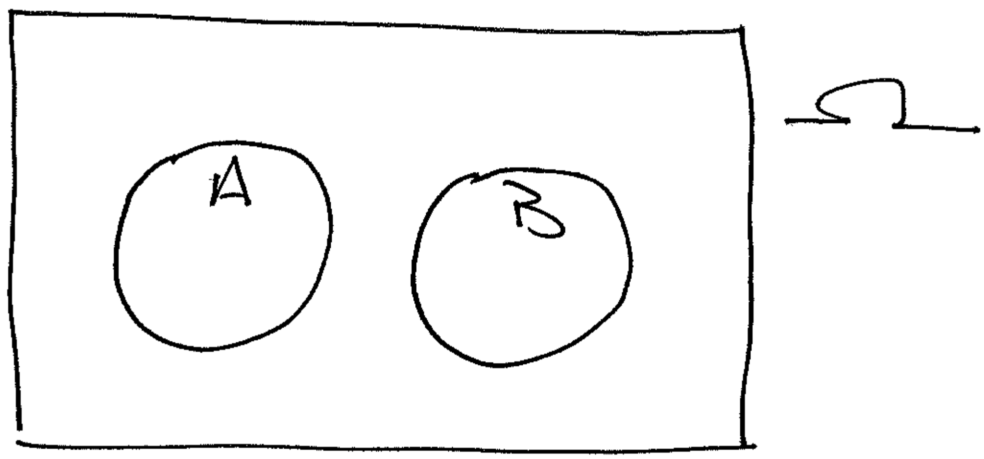
Axioms for $\mathbb{P}(\cdot)$

(i) $\mathbb{P}(A) \geq 0$ for all $A \subseteq \Omega$.

(ii) $\mathbb{P}(\Omega) = 1$

(iii) <u>additivity</u>

- if $A, B \subseteq \Omega$ are disjoint $(A \cap B = \phi)$, then
  - $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$



$$A \cap B = \emptyset$$

- <u>finite additivity</u>

  if $A_1, \dots, A_n \subseteq \Omega$ are pairwise disjoint ($A_i \cap A_j = \emptyset$ for any $i \neq j$), then

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$

- <u>Countable additivity</u>

if $A_1, A_2, \ldots\ldots$ are pairwise disjoint ($A_i \cap A_j = \phi$ if $i \neq j$),

then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

<u>Ex</u> flip a fair coin.

$$\Omega = \{H, T\}$$

<u>notation</u> $P(\{H\}) = P(H)$

$P(\{T\}) = P(T)$

fair means $\underline{P(H) = P(T)}$.

note

$$\underline{P(H) + P(T)} = P(\{H, T\})$$
$$= P(\Omega) = \underline{1}$$

since $\{H\} \cup \{T\} = \{H, T\}$ and
$\{H\} \cap \{T\} = \phi$. (by axiom iii)
(and axiom ii)

$\therefore$ $P(H) = P(T) = \frac{1}{2}$.

In General if $A \subseteq \Omega$,
then

$$A \cup A^c = \Omega$$
$$A \cap A^c = \phi$$

so by axiom (iii) and (ii)

$$P(A) + P(A^c) = 1 \qquad (*)$$

<u>Note</u> by axiom (i) $P(A) \geq 0$.
by $*$ $P(A) \leq 1$, thus

$$0 \leq P(A) \leq 1$$

for any $A \subseteq \Omega$.

Evidently P is a function

$$P : \mathbb{P}(\Omega) \longrightarrow [0, 1]$$

[<u>Note</u> for some sample spaces, some subsets $S \subseteq \Omega$ cannot be assigned a probability.]

<u>note</u>

$$
\begin{aligned}
1 &= P(\Omega) \\
&= P(\Omega \cup \Omega^c) \\
&= P(\Omega \cup \phi) \\
&= P(\Omega) + P(\phi) \\
&= 1 + P(\phi)
\end{aligned}
$$

$\therefore P(\phi) = 0$

Ex. fair coin (again)

$\Omega = \{H, T\}$

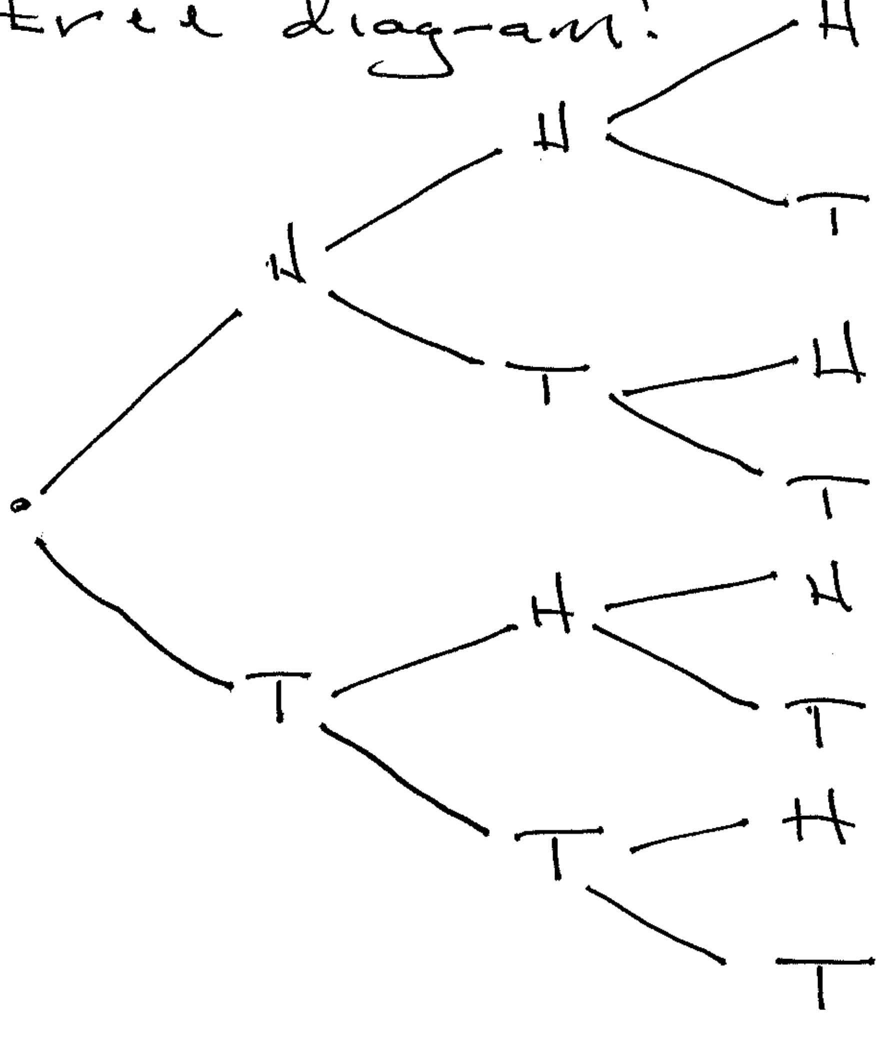
events: $\emptyset$, H, T, $\{H, T\}$

Probabilities: 0, $\frac{1}{2}$, $\frac{1}{2}$, 1

Ex. flip fair coin 3 times

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT,$
$TTH, TTT\}$

tree diagram:



Outcomes

HHH

HHT

HTH

HTT

THH

THT

TTH

TTT

coin 1   2   3

'fair' means equally likely, i.e.

$P(\text{outcome}) = \frac{1}{8}$.