cse 107 10-23-25

- no supplemental lecture this week
- no hw next week, hw4 will be due tuesday of week 6.
- lab 2 due tomorrow (Fri. 10/24)
- lab 3 posted.
- mid1 solutions posted soon

<u>Defn</u>

r.v.s $X, Y, Z$ are independent

iff

$$p_{X,Y,Z}(x,y,z) = p_X(x) \cdot p_Y(y) \cdot p_Z(z)$$

<u>Thus</u> if $X, Y, Z$ are independent, so are $f(X), g(Y), h(Z)$

Also if $X, Y, Z$ indep.,

then

- $E[X \cdot Y \cdot Z] = E[X] \cdot E[Y] \cdot E[Z]$
- $Var(X+Y+Z) = Var(X) + Var(Y) + Var(Z)$.

## Variance of the Binomial

Recall, if $X$ is Binomial$(n,p)$ then

$$X = X_1 + X_2 + \cdots + X_n$$

where $X_1, X_2, \ldots, X_n$ are identical, independent Bernoulli$(p)$

Thus

$$Var(X) = Var(X_1 + \cdots + X_n)$$

$$= Var(X_1) + \cdots + Var(X_n)$$

$$= p(1-p) + \cdots + p(1-p)$$

$$= np(1-p).$$

<u>Ex.</u>

Suppose we have a weighted coin, but we don't know P(head). Say

$$P(\text{head}) = p$$

To estimate $p$, flip coin $n$ times.

<u>Defn</u>

The <u>Sample Mean</u> is the r.v.

$$S_n = \frac{X_1 + \cdots + X_n}{n}$$

where $X_i = \begin{cases} 1 & \text{if head} \\ 0 & \text{if tail} \end{cases}$

Since $p$ is the $P(\text{head})$,
$E[X_i] = p$, so

$$E[S_n] = E\left[\frac{1}{n}X_1 + \cdots + \frac{1}{n}X_n\right]$$

$$= \frac{1}{n}E[X_1] + \cdots + \frac{1}{n}E[X_n]$$

$$= \frac{1}{n}(p + \cdots + p)$$

$$= \frac{1}{\cancel{n}} \cdot \cancel{n}p = p$$

$\therefore S_n$ is an estimate of $p$.

Why?