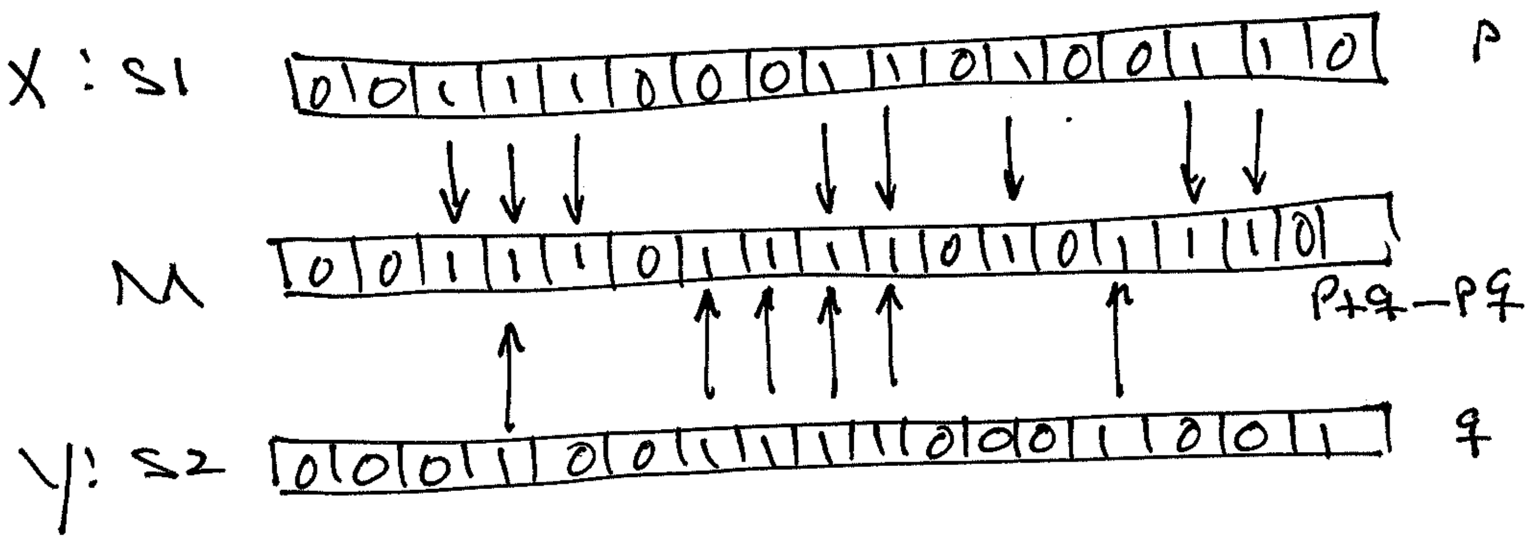
CSE 107 11-21-24

Merging two Bernoulli Processes



Rule:

$$M_t = \begin{cases} 1 & \text{if either } X_t = 1 \ \underline{\underline{\text{or}}} \ Y_t = 1 \\ 0 & \text{otherwise} \end{cases}$$

Observe

$$P(M_t = 1) = 1 - P(M_t = 0)$$

$$= 1 - P(X_t = 0, Y_t = 0)$$

$$= 1 - P(X_t = 0) \cdot P(Y_t = 0)$$

$$= 1 - (1-p)(1-q)$$

$$= \cancel{1} - (\cancel{1} - p - q + pq)$$

$$= p + q - pq$$

This is a Bernoulli sequence!

Exercises

- first split, then merge. determine the Parameter.
- first merge, then split. find Parameter.
- change bit operation in merge, from <u>or</u> to <u>and</u>

  <u>exor</u>

  <u>nand</u>

  <u>nor</u>

6.2 The Poisson Process

Summary of Bernoulli Process:

seq. of Bernoulli R.V.s

$$\{X_t \mid t = 1, 2, 3, \ldots\}$$

(1) X_t is Bernoulli with Param p

(2) the set $\{X_t \mid t = 1, 2, \ldots\}$ is independent. i.e. events occuring in disjoint time intervals are indep.

Defined

- Y_k = time to k^{th} arrival

  Pascal of order k

- $T_1 = Y_1$

  $\vdots$

  $T_k = Y_k - Y_{k-1}$

  Geometric, Param. p

We can also define

$$N_t = X_1 + X_2 + \cdots + X_t \quad (t = 1, 2, ..)$$

$$= \# \text{ of arrivals in } [1, t]$$

so N_t is Binomial with

Parameters t, p :

$$P_{N_t}(k) = \binom{t}{k} \cdot p^k (1-p)^{t-k} \quad (k = 1, 2, \ldots)$$